# Chapter 10

# Electric Circuits - Grade 10

## 10.1 Electric Circuits

In South Africa, people depend on electricity to provide power for most appliances in the home, at work and out in the world in general. For example, flourescent lights, electric heating and cooking (on electric stoves), all depend on electricity to work. To realise just how big an impact electricity has on our daily lives, just think about what happens when there is a power failure or load shedding.

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**Activity :: Discussion : Uses of electricity**
With a partner, take the following topics and, for each topic, write down at least 5 items/appliances/machines which need electricity to work. Try not to use the same item more than once.

- At home
- At school
- At the hospital
- In the city

Once you have finished making your lists, compare with the lists of other people in your class. (Save your lists somewhere safe for later because there will be another activity for which you'll need them.)
When you start comparing, you should notice that there are many different items which we use in our daily lives which rely on electricity to work!

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**Important: Safety Warning:** We believe in experimenting and learning about physics at every opportunity, BUT playing with electricity can be **EXTREMELY DANGEROUS**! Do not try to build home made circuits alone. Make sure you have someone with you who knows if what you are doing is safe. Normal electrical outlets are dangerous. Treat electricity with respect in your everyday life.

### 10.1.1 Closed circuits

In the following activity we will investigate what is needed to cause charge to flow in an electric circuit.

**Activity :: Experiment : Closed circuits**

**Aim:**
To determine what is required to make electrical charges flow. In this experiment, we will use a lightbulb to check whether electrical charge is flowing in the circuit or not. If charge is flowing, the lightbulb should glow. On the other hand, if no charge is flowing, the lightbulb will not glow.

**Apparatus:**
You will need a small lightbulb which is attached to a metal conductor (e.g. a bulb from a school electrical kit), some connecting leads and a battery.

**Method:**
Take the apparatus items and try to connect them in a way that you cause the light bulb to glow (i.e. charge flows in the circuit).

**Questions:**

1. Once you have arranged your circuit elements to make the lightbulb glow, draw your circuit.
2. What can you say about how the battery is connected? (i.e. does it have one or two connecting leads attached? Where are they attached?)
3. What can you say about how the light bulb is connected in your circuit? (i.e. does it connect to one or two connecting leads, and where are they attached?)
4. Are there any items in your circuit which are not attached to something? In other words, are there any gaps in your circuit?

Write down your conclusion about what is needed to make an electric circuit work and charge to flow.

In the experiment above, you will have seen that the light bulb only glows when there is a *closed* circuit i.e. there are no gaps in the circuit and all the circuit elements are connected in a *closed loop*. Therefore, in order for charges to flow, a closed circuit and an energy source (in this case the battery) are needed. (Note: you do not have to have a lightbulb in the circuit! We used this as a check that charge was flowing.)

**Definition: Electric circuit**
An electric circuit is a closed path (with no breaks or gaps) along which electrical charges (electrons) flow powered by an energy source.

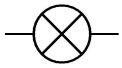
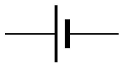
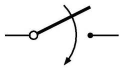
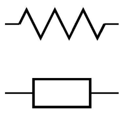
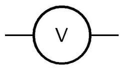
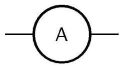
### 10.1.2 Representing electric circuits

#### Components of electrical circuits

Some common elements (components) which can be found in electrical circuits include light bulbs, batteries, connecting leads, switches, resistors, voltmeters and ammeters. You will learn more about these items in later sections, but it is important to know what their symbols are and how to represent them in circuit diagrams. Below is a table with the items and their symbols:

#### Circuit diagrams

**Definition: Representing circuits**
A **physical circuit** is the electric circuit you create with real components.
A **circuit diagram** is a drawing which uses symbols to represent the different components in the physical circuit.

| Component | Symbol | Usage |
|---|---|---|
| light bulb |  | glows when charge moves through it |
| battery |  | provides energy for charge to move |
| switch |  | allows a circuit to be open or closed |
| resistor |  | resists the flow of charge |
| voltmeter |  | measures potential difference |
| ammeter |  | measures current in a circuit |
| connecting lead | | connects circuit elements together |

We use circuit diagrams to represent circuits because they are much simpler and more general than drawing the physical circuit because they only show the workings of the electrical components. You can see this in the two pictures below. The first picture shows the *physical circuit* for an electric torch. You can see the light bulb, the batteries, the switch and the outside plastic casing of the torch. The picture is actually a *cross-section* of the torch so that we can see inside it.

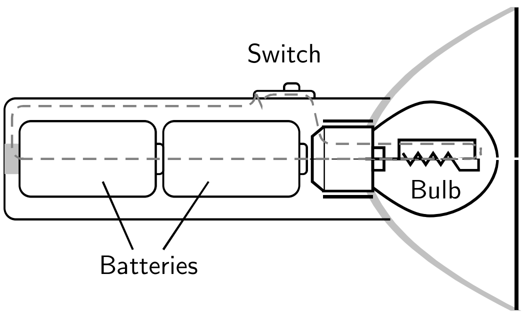



Figure 10.1: Physical components of an electric torch. The dotted line shows the path of the electrical circuit.

Below is the *circuit diagram* for the electric torch. Now the light bulb is represented by its symbol, as are the batteries, the switch and the connecting wires. It is not necessary to show the plastic casing of the torch since it has nothing to do with the electric workings of the torch. You can see that the circuit diagram is much simpler than the physical circuit drawing!

### Series and parallel circuits

There are two ways to connect electrical components in a circuit: **in series** or **in parallel**.

**Definition: Series circuit**
In a series circuit, the charge has a **single** path from the battery, returning to the battery.

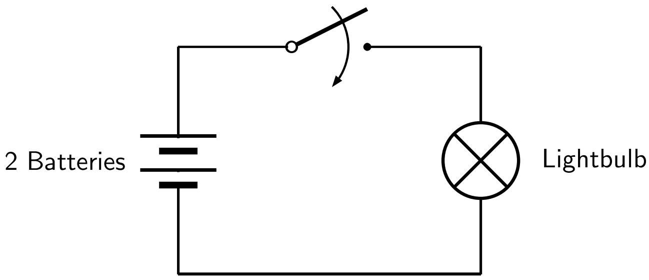
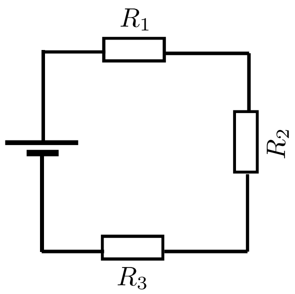


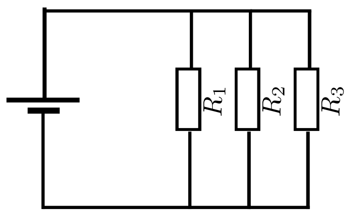

Figure 10.2: Circuit diagram of an electric torch.

**Definition: Parallel circuit**
In a parallel circuit, the charge has **multiple** paths from the battery, returning to the battery.

The picture below shows a circuit with three resistors connected *in series* on the left and a circuit with three resistors connected *in parallel* on the right:




3 resistors in a series circuit




3 resistors in a parallel circuit

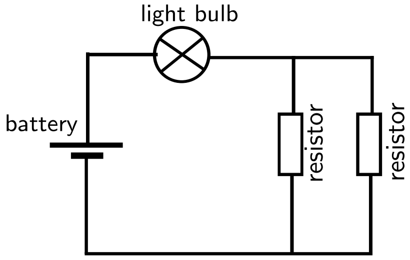
**Worked Example 43: Drawing circuits I**

**Question:** Draw the circuit diagram for a circuit which has the following components:

1. 1 battery
2. 1 lightbulb connected in series
3. 2 resistors connected in parallel

**Answer**

**Step 1 : Identify the components and their symbols and draw according to the instructions:**



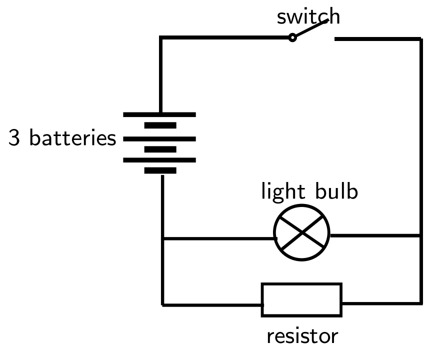
**Worked Example 44: Drawing circuits II**

**Question:** Draw the circuit diagram for a circuit which has the following components:

1. 3 batteries in series
2. 1 lightbulb connected in parallel with 1 resistor
3. a switch in series

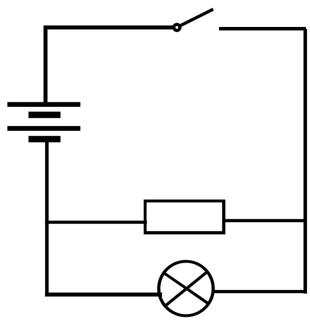
**Answer**

**Step 1 : Identify the symbol for each component and draw according to the instructions:**




**Exercise: Circuits**

1. Using physical components, set up the physical circuit which is described by the circuit diagram below:



   1.1 Now draw a picture of the physical circuit you have built.

2. Using physical components, set up a closed circuit which has one battery and a light bulb in series with a resistor.
   2.1 Draw the physical circuit.
   2.2 Draw the resulting circuit diagram.
   2.3 How do you know that you have built a closed circuit? (What happens to the light bulb?)
   2.4 If you add one more resistor to your circuit (also in series), what do you notice? (What happens to the light from the light bulb?)
   2.5 Draw the new circuit diagram which includes the second resistor.

3. Draw the circuit diagram for the following circuit: 2 batteries, a switch in series and 1 lightbulb which is in parallel with two resistors.
   3.1 Now use physical components to set up the circuit.

3.2 What happens when you close the switch? What does does this mean about the circuit?
3.3 Draw the physical circuit.

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**Activity :: Discussion : Alternative Energy**

At the moment, electric power is produced by burning fossil fuels such as coal and oil. In South Africa, our main source of electric power is coal burning power stations. (We also have one nuclear power plant called Koeberg in the Western Cape). However, burning fossil fuels releases large amounts of pollution into the earth's atmosphere and can contribute to global warming. Also, the earth's fossil fuel reserves (especially oil) are starting to run low. For these reasons, people all across the world are working to find *alternative*/other sources of energy and on ways to *conserve*/save energy. Other sources of energy include wind power, solar power (from the sun), hydro-electric power (from water) among others.

With a partner, take out the lists you made earlier of the item/appliances/machines which used electricity in the following environments. For each item, try to think of an *alternative* AND a way to *conserve* or save power.

For example, if you had a flourescent light as an item used in the home, then:

- Alternative: use candles at supper time to reduce electricity consumption
- Conservation: turn off lights when not in a room, or during the day.

**Topics:**

- At home
- At school
- At the hospital
- In the city

Once you have finished making your lists, compare with the lists of other people in your class.

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## 10.2 Potential Difference

### 10.2.1 Potential Difference

When a circuit is connected and is a complete circuit charge can move through the circuit. Charge will not move unless there is a reason, a force. Think of it as though charge is at rest and something has to push it along. This means that work needs to be done to make charge move. A force acts on the charges, doing work, to make them move. The force is provided by the battery in the circuit.

We call the moving charge "current" and we will talk about this later.

The position of the charge in the circuit tells you how much potential energy it has because of the force being exerted on it. This is like the force from gravity, the higher an object is above the ground (position) the more potential energy it has.

The amount of work to move a charge from one point to another point is how much the potential energy has changed. This is the difference in potential energy, called potential difference. Notice that it is a difference between the value of potential energy at two points so we say that potential difference is measured between or across two points. We do not say potential difference through something.

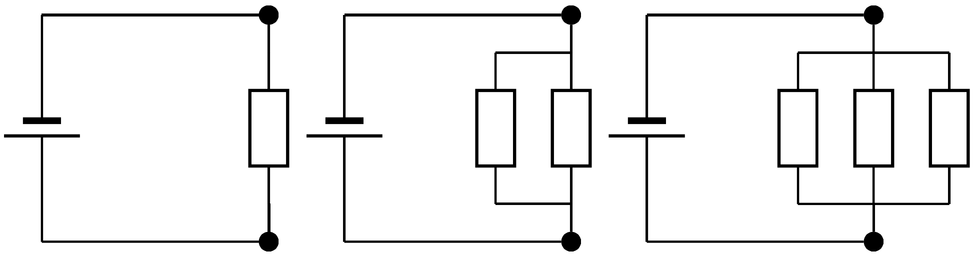
**Definition: Potential Difference**
Electrical potential difference as the difference in electrical potential energy per unit charge between two points. The units of potential difference are the volt (V).

The units are volt (V), which is the same as joule per coulomb, the amount of work done per unit charge. Electrical potential difference is also called voltage.

### 10.2.2 Potential Difference and Parallel Resistors

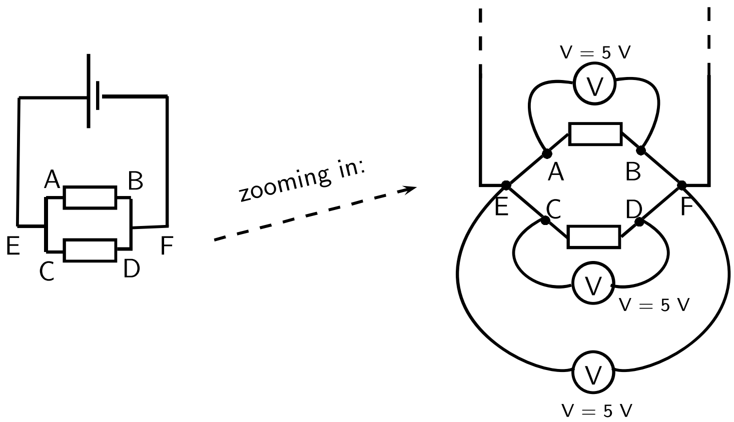
When resistors are connected in parallel the start and end points for all the resistors are the same. These points have the same potential energy and so the potential difference between them is the same no matter what is put in between them. You can have one, two or many resistors between the two points, the potential difference will not change. You can ignore whatever components are between two points in a circuit when calculating the difference between the two points.

Look at the following circuit diagrams. The battery is the same in all cases, all that changes is more resistors are added between the points marked by the black dots. If we were to measure the potential difference between the two dots in these circuits we would get the same answer for all three cases.



Lets look at two resistors in parallel more closely. When you construct a circuit you use wires and you might think that measuring the voltage in different places on the wires will make a difference. This is not true. The potential difference or voltage measurement will only be different if you measure a different set of components. All points on the wires that have no circuit components between them will give you the same measurements.

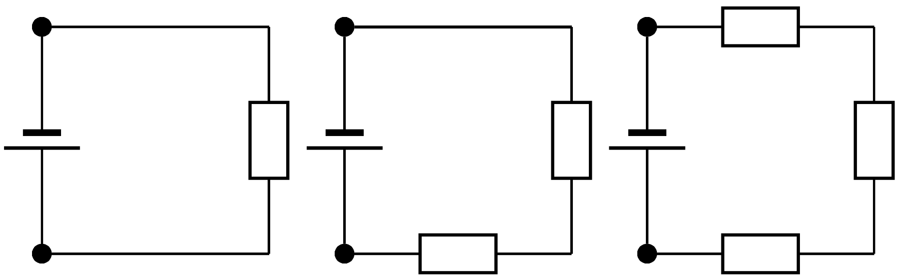
All three of the measurements shown in the picture below will give you the same voltages. The different measurement points on the left have no components between them so there is no change in potential energy. Exactly the same applies to the different points on the right. When you measure the potential difference between the points on the left and right you will get the same answer.



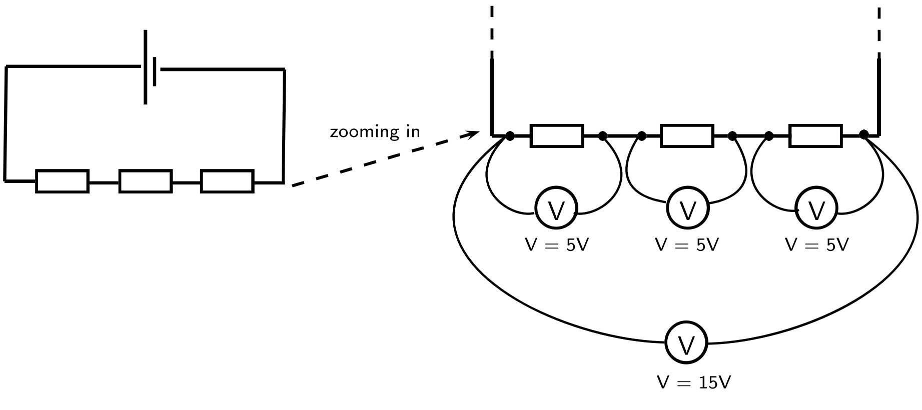
### 10.2.3 Potential Difference and Series Resistors

When resistors are in series, one after the other, there is a potential difference across each resistor. The total potential difference across a set of resistors in series is the sum of the potential differences across each of the resistors in the set. This is the same as falling a large distance under gravity or falling that same distance (difference) in many smaller steps. The total distance (difference) is the same.

Look at the circuits below. If we measured the potential difference between the black dots in all of these circuits it would be the same just like we saw above. So we now know the total potential difference is the same across one, two or three resistors. We also know that some work is required to make charge flow through each one, each is a step down in potential energy. These steps add up to the total drop which we know is the difference between the two dots.

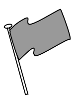


Let us look at this in a bit more detail. In the picture below you can see what the different measurements for 3 identical resistors in series could look like. The total voltage across all three resistors is the sum of the voltages across the individual resistors.



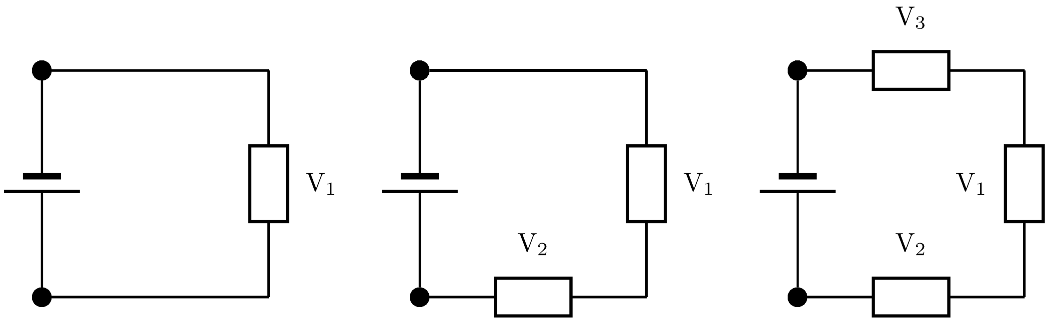

### 10.2.4 Ohm's Law

The voltage is the change in potential energy or work done when charge moves between two points in the circuit. The greater the resistance to charge moving the more work that needs to be done. The work done or voltage thus depends on the resistance. The potential difference is proportional to the resistance.



**Definition: Ohm's Law**
Voltage across a circuit component is proportional to the resistance of the component.

Use the fact that voltage is proportional to resistance to calculate what proportion of the total voltage of a circuit will be found across each circuit element.




We know that the total voltage is equal to $V_1$ in the first circuit, to $V_1 + V_2$ in the second circuit and $V_1 + V_2 + V_3$ in the third circuit.

We know that the potential energy lost across a resistor is proportional to the resistance of the component. The total potential difference is shared evenly across the total resistance of the circuit. This means that the potential difference per unit of resistance is

$$V_{per\ unit\ of\ resistance} = \frac{V_{total}}{R_{total}}$$

Then the voltage across a resistor is just the resistance times the potential difference per unit of resistance

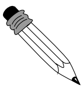
$$V_{resistor} = R_{resistor} \cdot \frac{V_{total}}{R_{total}}.$$

### 10.2.5 EMF

When you measure the potential difference across (or between) the terminals of a battery you are measuring the "electromotive force" (emf) of the battery. This is how much potential energy the battery has to make charges move through the circuit. This driving potential energy is equal to the total potential energy drops in the circuit. This means that the voltage across the battery is equal to the sum of the voltages in the circuit.

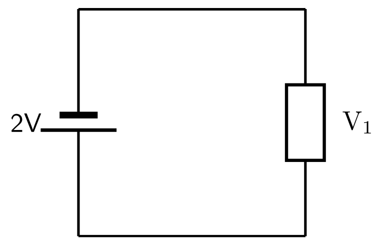
We can use this information to solve problems in which the voltages across elements in a circuit add up to the emf.

$$EMF = V_{total}$$



**Worked Example 45: Voltages I**

**Question:** What is the voltage across the resistor in the circuit shown?




**Answer**

**Step 1 : Check what you have and the units**

We have a circuit with a battery and one resistor. We know the voltage across the battery. We want to find that voltage across the resistor.

$$V_{battery} = 2\text{V}$$

**Step 2 : Applicable principles**

We know that the voltage across the battery must be equal to the total voltage across all other circuit components.

$$V_{battery} = V_{total}$$

There is only one other circuit component, the resistor.

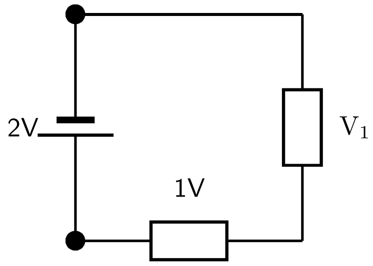
$$V_{total} = V_1$$

This means that the voltage across the battery is the same as the voltage across the resistor.

$$V_{battery} = V_{total} = V_1$$
$$V_{battery} = V_{total} = V_1$$
$$V_1 = 2\text{V}$$

**Worked Example 46: Voltages II**

**Question:** What is the voltage across the unknown resistor in the circuit shown?




**Answer**

**Step 1 : Check what you have and the units**

We have a circuit with a battery and two resistors. We know the voltage across the battery and one of the resistors. We want to find that voltage across the resistor.

$$V_{battery} = 2\text{V}$$

$$V_{resistor} = 1\text{V}$$

**Step 2 : Applicable principles**

We know that the voltage across the battery must be equal to the total voltage across all other circuit components.

$$V_{battery} = V_{total}$$

The total voltage in the circuit is the sum of the voltages across the individual resistors

$$V_{total} = V_1 + V_{resistor}$$

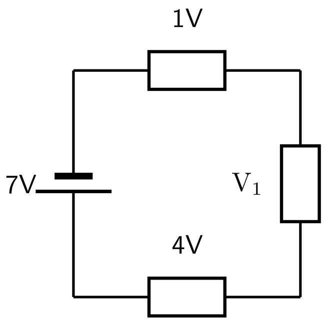
Using the relationship between the voltage across the battery and total voltage across the resistors

$$V_{battery} = V_{total}$$

$$\begin{aligned} V_{battery} &= V_1 + V_{resistor} \\ 2\text{V} &= V_1 + 1\text{V} \\ V_1 &= 1\text{V} \end{aligned}$$

**Worked Example 47: Voltages III**

**Question:** What is the voltage across the unknown resistor in the circuit shown?




**Answer**

**Step 1 : Check what you have and the units**

We have a circuit with a battery and three resistors. We know the voltage across the battery and two of the resistors. We want to find that voltage across the unknown resistor.

$$V_{battery} = 7\text{V}$$

$$V_{known} = 1\text{V} + 4\text{V}$$

**Step 2 : Applicable principles**

We know that the voltage across the battery must be equal to the total voltage across all other circuit components.

$$V_{battery} = V_{total}$$

The total voltage in the circuit is the sum of the voltages across the individual resistors

$$V_{total} = V_1 + V_{known}$$

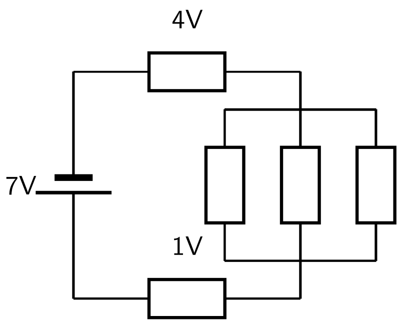
Using the relationship between the voltage across the battery and total voltage across the resistors

$$V_{battery} = V_{total}$$

$$\begin{aligned} V_{battery} &= V_1 + V_{known} \\ 7\text{V} &= V_1 + 5\text{V} \\ V_1 &= 2\text{V} \end{aligned}$$

**Worked Example 48: Voltages IV**

**Question:** What is the voltage across the parallel resistor combination in the circuit shown? Hint: the rest of the circuit is the same as the previous problem.




**Answer**

**Step 1 : Quick Answer**

The circuit is the same as the previous example and we know that the voltage difference between two points in a circuit does not depend on what is between them so the answer is the same as above $V_{parallel} = 2\text{V}$.

**Step 2 : Check what you have and the units - long answer**

We have a circuit with a battery and three resistors. We know the voltage across the battery and two of the resistors. We want to find that voltage across the parallel resistors, $V_{parallel}$.

$$V_{battery} = 7\text{V}$$

$$V_{known} = 1\text{V} + 4\text{V}$$

**Step 3 : Applicable principles**

We know that the voltage across the battery must be equal to the total voltage across all other circuit components.

$$V_{battery} = V_{total}$$

The total voltage in the circuit is the sum of the voltages across the individual resistors

$$V_{total} = V_{parallel} + V_{known}$$

Using the relationship between the voltage across the battery and total voltage across the resistors

$$V_{battery} = V_{total}$$

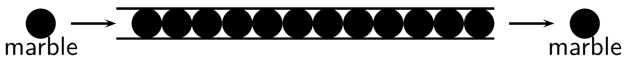
$$\begin{aligned} V_{battery} &= V_{parallel} + V_{known} \\ 7\text{V} &= V_1 + 5\text{V} \\ V_{parallel} &= 2\text{V} \end{aligned}$$

## 10.3 Current

### 10.3.1 Flow of Charge

We have been talking about moving charge. We need to be able to deal with numbers, how much charge is moving, how fast is it moving? The concept that gives us this information is called *current*. Current allows us to quantify the movement of charge.

When we talk about current we talk about how much charge moves past a fixed point in circuit in one second. Think of charges being pushed around the circuit by the battery, there are charges in the wires but unless there is a battery they won't move. When one charge moves the charges next to it also move. They keep their spacing. If you had a tube of marbles like in this picture.




If you push one marble into the tube one must come out the other side. If you look at any point in the tube and push one marble into the tube, one marble will move past the point you are looking at. This is similar to charges in the wires of a circuit.

If a charge moves they all move and the same number move at every point in the circuit.

### 10.3.2 Current

Now that we've thought about the moving charges and visualised what is happening we need to get back to quantifying moving charge. I've already told you that we use current but we still need to define it.

**Definition: Current**

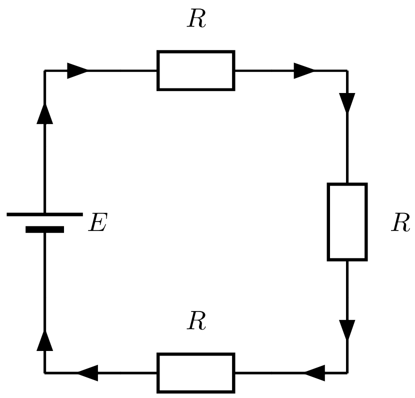
Current is the rate at which charges moves past a fixed point in a circuit. We use the symbol I to show current and it is measured in amperes (A). One ampere is one coulomb of charge moving in one second.

$$I = \frac{Q}{\Delta t}$$

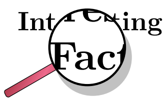
When current flows in a circuit we show this on a diagram by adding arrows. The arrows show the direction of flow in a circuit. By convention we say that charge flows from the positive terminal on a battery to the negative terminal.

### 10.3.3 Series Circuits

In a series circuit, the charge has a single path from the battery, returning to the battery.




The arrows in this picture show you the direction that charge will flow in the circuit. They don't show you much charge will flow, only the direction.




Benjamin Franklin made a guess about the direction of charge flow when rubbing smooth wax with rough wool. He thought that the charges flowed from the wax to the wool (i.e. from positive to negative) which was opposite to the real direction. Due to this, electrons are said to have a *negative* charge and so objects which Ben Franklin called "negative" (meaning a shortage of charge) really have an excess of electrons. By the time the true direction of electron flow was discovered, the convention of "positive" and "negative" had already been so well accepted in the scientific world that no effort was made to change it.

**Important:** A cell **does not** produce the same amount of current no matter what is connected to it. While the voltage produced by a cell is constant, the amount of current supplied depends on what is in the circuit.

How does the current through the battery in a circuit with several resistors in series compare to the current in a circuit with a single resistor?

---

**Activity :: Experiment : Current in Series Circuits**

**Aim:**

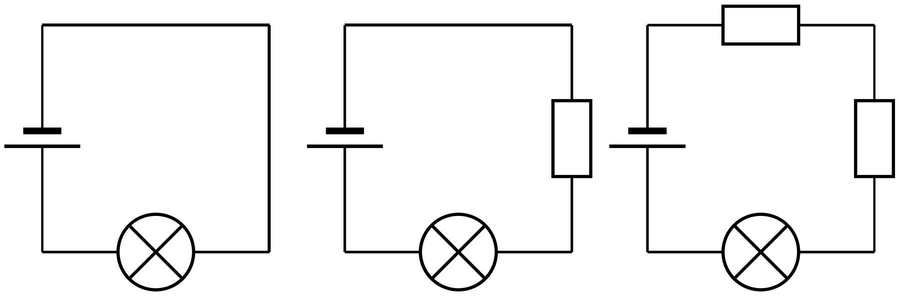
To determine the effect of multiple resistors on current in a circuit

**Apparatus:**

- Battery
- Resistors
- Light bulb
- Wires

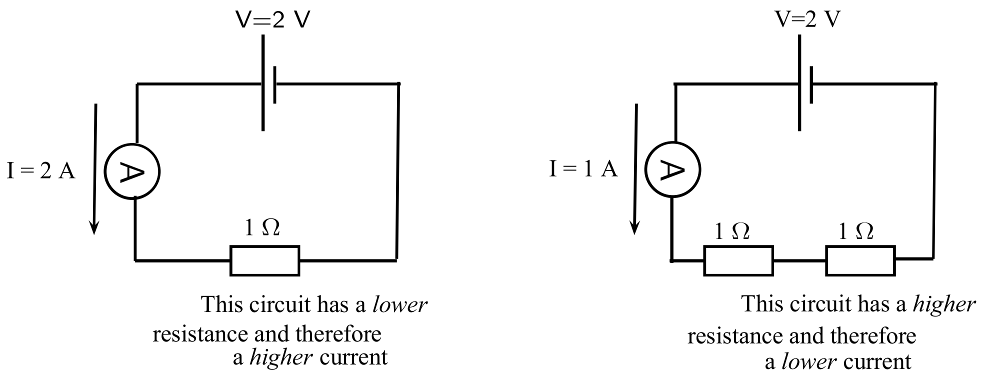
**Method:**

1. Construct the following circuits

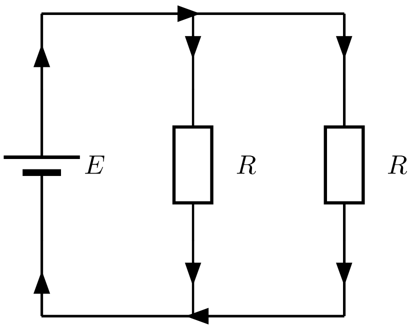


2. Rank the three circuits in terms of the brightness of the bulb.

**Conclusions:**
The brightness of the bulb is an indicator of how much current is flowing. If the bulb gets brighter because of a change then more current is flowing. If the bulb gets dimmer less current is flowing. You will find that the more resistors you have the dimmer the bulb.




### 10.3.4 Parallel Circuits




How does the current through the battery in a circuit with several resistors in parallel compare to the current in a circuit with a single resistor?

**Activity :: Experiment : Current in Series Circuits**

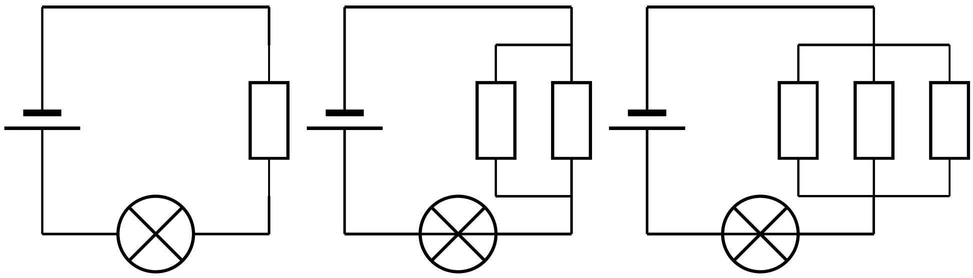
**Aim:**
To determine the effect of multiple resistors on current in a circuit

**Apparatus:**

- Battery
- Resistors
- Light bulb
- Wires

**Method:**

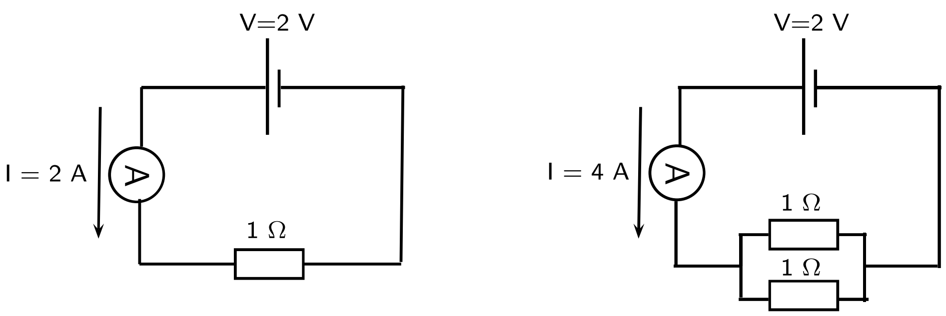
1. Construct the following circuits



2. Rank the three circuits in terms of the brightness of the bulb.

**Conclusions:**
The brightness of the bulb is an indicator of how much current is flowing. If the bulb gets brighter because of a change then more current is flowing. If the bulb gets dimmer less current is flowing. You will find that the more resistors you have the brighter the bulb.

Why is this the case? Why do more resistors make it easier for charge to flow in the circuit? It is because they are in parallel so there are more paths for charge to take to move. You can think of it like a highway with more lanes, or the tube of marbles splitting into multiple parallel tubes. The more branches there are, the easier it is for charge to flow. You will learn more about the total resistance of parallel resistors later but always remember that more resistors in parallel mean more pathways. In series the pathways come one after the other so it does not make it easier for charge to flow.




the 2 resistors in parallel result in a *lower* total resistance and therefore a *higher* current in the circuit

## 10.4 Resistance

### 10.4.1 What causes resistance?

We have spoken about resistors that slow down the flow of charge in a conductor. On a microscopic level, electrons moving through the conductor collide with the particles of which the conductor (metal) is made. When they collide, they transfer kinetic energy. The electrons therefore lose kinetic energy and slow down. This leads to resistance. The transferred energy causes the conductor to heat up. You can feel this directly if you touch a cellphone charger when you are charging a cell phone - the charger gets warm!

**Definition: Resistance**
Resistance slows down the flow of charge in a circuit. We use the symbol **R** to show resistance and it is measured in units called **Ohms** with the symbol $\Omega$.

$$1 \text{ Ohm} = 1 \frac{\text{Volt}}{\text{Ampere}}.$$

*All* conductors have some resistance. For example, a piece of wire has less resistance than a light bulb, but both have resistance. The high resistance of the filament (small wire) in a lightbulb causes the electrons to transfer a lot of their kinetic energy in the form of heat. The heat energy is enough to cause the filament to glow white-hot which produces light. The wires connecting the lamp to the cell or battery hardly even get warm while conducting the same amount of current. This is because of their much lower resistance due to their larger cross-section (they are thicker).

An important effect of a resistor is that it *converts* electrical energy into other forms of energy, such as **heat** and **light**.

Interesting Fact

There is a special type of conductor, called a **superconductor** that has no resistance, but the materials that make up superconductors only start superconducting at very low temperatures (approximately -170°C).

#### Why do batteries go flat?

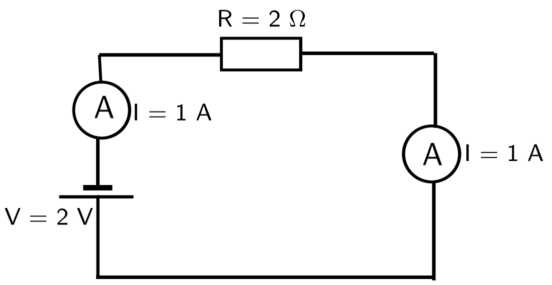
A battery stores chemical potential energy. When it is connected in a circuit, a chemical reaction takes place inside the battery which converts chemical potential energy to electrical energy which powers the electrons to move through the circuit. All the circuit elements (such as the conducting leads, resistors and lightbulbs) have some resistance to the flow of charge and convert the electrical energy to heat and/or light. The battery goes flat when all its chemical potential energy has been converted into other forms of energy.

### 10.4.2 Resistors in electric circuits

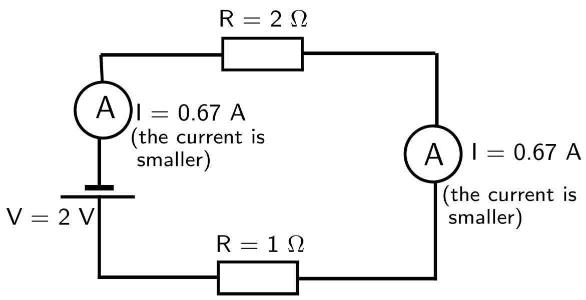
It is important to understand what effect adding resistors to a circuit has on the *total* resistance of a circuit and on the current that can flow in the circuit.

### Resistors in series

When we add resistors in series to a circuit, we *increase* the resistance to the flow of current. There is only **one path** that the current can flow down and the current is the same at all places in the series circuit. Take a look at the diagram below: On the left there is a circuit with a single resistor and a battery. No matter where we measure the current, it is the same in a series circuit. On the right, we have added a second resistor in series to the circuit. The *total* resistance of the circuit has *increased* and you can see from the reading on the ammeter that the current in the circuit has decreased.




The current in a series circuit is the same everywhere

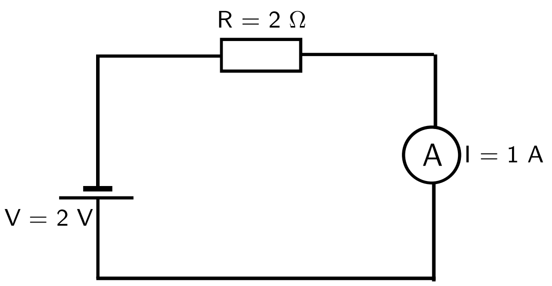


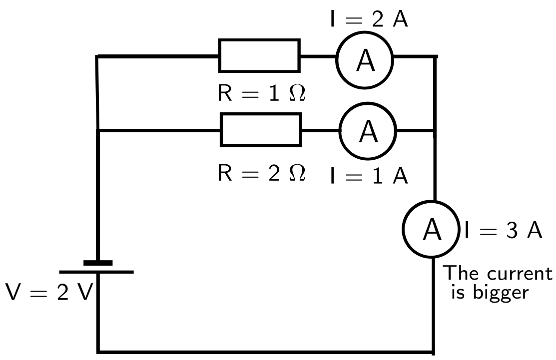

Adding a resistor to the circuit increases the total resistance

### Resistors in parallel

In contrast to the series case, when we add resistors in parallel, we create **more paths** along which current can flow. By doing this we *decrease* the total resistance of the circuit!

Take a look at the diagram below. On the left we have the same circuit as in the previous diagram with a battery and a resistor. The ammeter shows a current of 1 ampere. On the right we have added a second resistor in parallel to the first resistor. This has increased the number of paths (branches) the charge can take through the circuit - the total resistance has decreased. You can see that the current in the circuit has increased. Also notice that the current in the different branches can be different.




Adding a resistor to the circuit in parallel decreases the total resistance

---

**Exercise: Resistance**

1. What is the unit of resistance called and what is its symbol?
2. Explain what happens to the total resistance of a circuit when resistors are added in series?

3. Explain what happens to the total resistance of a circuit when resistors are added in parallel?
4. Why do batteries go flat?

## 10.5 Instruments to Measure voltage, current and resistance

As we have seen in previous sections, an electric circuit is made up of a number of different components such as batteries, resistors and light bulbs. There are devices to measure the properties of these components. These devices are called meters.

For example, one may be interested in measuring the amount of current flowing through a circuit using an *ammeter* or measuring the voltage provided by a battery using a *voltmeter*. In this section we will discuss the practical usage of voltmeters, ammeters, and *ohmmeters*.

### 10.5.1 Voltmeter

A voltmeter is an instrument for measuring the voltage between two points in an electric circuit. In analogy with a water circuit, a voltmeter is like a meter designed to measure pressure difference. Since one is interested in measuring the voltage between two points in a circuit, a voltmeter must be connected in *parallel* with the portion of the circuit on which the measurement is made.

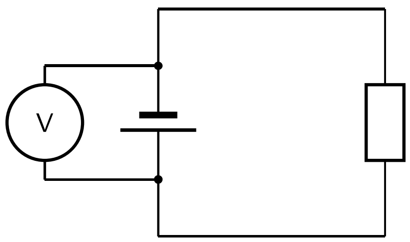



Figure 10.3: A voltmeter should be connected in parallel in a circuit.

Figure 10.3 shows a voltmeter connected in parallel with a battery. One lead of the voltmeter is connected to one end of the battery and the other lead is connected to the opposite end. The voltmeter may also be used to measure the voltage across a resistor or any other component of a circuit that has a voltage drop.

### 10.5.2 Ammeter

An ammeter is an instrument used to measure the flow of electric current in a circuit. Since one is interested in measuring the current flowing *through* a circuit component, the ammeter must be connected in *series* with the measured circuit component (Figure 10.4).

### 10.5.3 Ohmmeter

An ohmmeter is an instrument for measuring electrical resistance. The basic ohmmeter can function much like an ammeter. The ohmmeter works by suppling a constant voltage to the resistor and measuring the current flowing through it. The measured current is then converted into a corresponding resistance reading through Ohm's Law. Ohmmeters only function correctly when measuring resistance that is not being powered by a voltage or current source. In other words, you cannot measure the resistance of a component that is already connected to a circuit. This is because the ohmmeter's accurate indication depends only on its own source of voltage.

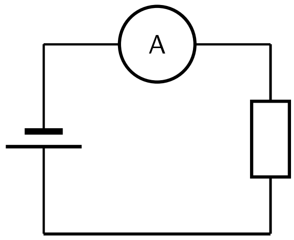



Figure 10.4: An ammeter should be connected in series in a circuit.

The presence of **any other** voltage across the measured circuit component interferes with the ohmmeter's operation. Figure 10.5 shows an ohmmeter connected with a resistor.

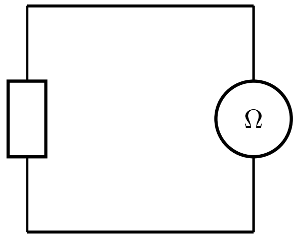



Figure 10.5: An ohmmeter should be used outside when there are no voltages present in the circuit.

### 10.5.4 Meters Impact on Circuit

A good quality meter used correctly will not significantly change the values it is used to measure. This means that an ammeter has very low resistance to not slow down the flow of charge.

A voltmeter has a very high resistance so that it does not add another parallel pathway to the circuit for the charge to flow along.

---

**Activity :: Investigation : Using meters**
If possible, connect meters in circuits to get used to the use of meters to measure electrical quantities. If the meters have more than one scale, always connect to the **largest scale** first so that the meter will not be damaged by having to measure values that exceed its limits.

---

The table below summarises the use of each measuring instrument that we discussed and the way it should be connected to a circuit component.

| Instrument | Measured Quantity | Proper Connection |
|---|---|---|
| Voltmeter | Voltage | In Parallel |
| Ammeter | Current | In Series |
| Ohmmeter | Resistance | Only with Resistor |

## 10.6 Exercises - Electric circuits

1. Write definitions for each of the following:

1.1 resistor
1.2 coulomb
1.3 voltmeter

2. Draw a circuit diagram which consists of the following components:
   2.1 2 batteries in parallel
   2.2 an open switch
   2.3 2 resistors in parallel
   2.4 an ammeter measuring total current
   2.5 a voltmeter measuring potential difference across one of the parallel resistors

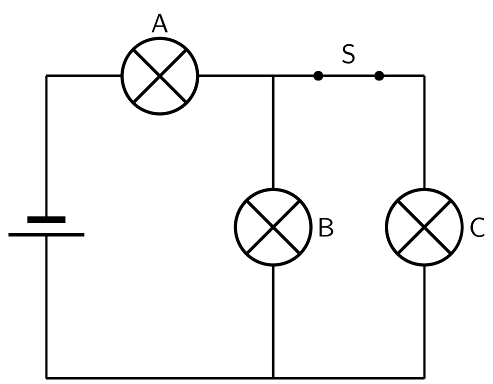
3. Complete the table below:

| Quantity | Symbol | Unit of meaurement | Symbol of unit |
|---|---|---|---|
| e.g. Distance | e.g. d | e.g. kilometer | e.g. km |
| Resistance | | | |
| Current | | | |
| Potential difference | | | |

4. [SC 2003/11] The emf of a battery can best be explained as the . . .
   4.1 rate of energy delivered per unit current
   4.2 rate at which charge is delivered
   4.3 rate at which energy is delivered
   4.4 charge per unit of energy delivered by the battery

5. [IEB 2002/11 HG1] Which of the following is the correct definition of the emf of a cell?
   5.1 It is the product of current and the external resistance of the circuit.
   5.2 It is a measure of the cell's ability to conduct an electric current.
   5.3 It is equal to the "lost volts" in the internal resistance of the circuit.
   5.4 It is the power dissipated per unit current passing through the cell.

6. [IEB 2005/11 HG] Three identical light bulbs A, B and C are connected in an electric circuit as shown in the diagram below.




How do the currents in bulbs A and B change when switch S is opened?

| | Current in A | Current in B |
|---|---|---|
| (a) | decreases | increases |
| (b) | decreases | decreases |
| (c) | increases | increases |
| (d) | increases | decreases |

# Chapter 19

# Electric Circuits - Grade 11

## 19.1 Introduction

The study of electrical circuits is essential to understand the technology that uses electricity in the real-world. This includes electricity being used for the operation of electronic devices like computers.

## 19.2 Ohm's Law

### 19.2.1 Definition of Ohm's Law

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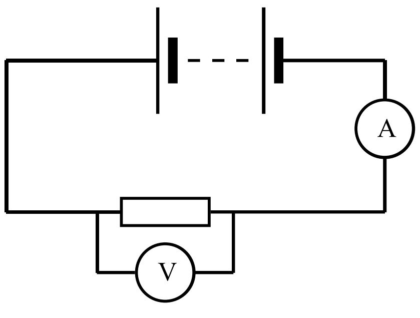
**Activity :: Experiment : Ohm's Law**
**Aim:**
In this experiment we will look at the relationship between the current going through a resistor and the potential difference (voltage) across the same resistor.




**Method:**

1. Set up the circuit according to the circuit diagram.
2. Draw the following table in your lab book.

| Voltage, $V$ (V) | Current, $I$ (A) |
|---|---|
| 1,5 | |
| 3,0 | |
| 4,5 | |
| 6,0 | |

3. Get your teacher to check the circuit before turning the power on.
4. Measure the current.
5. Add one more 1,5 V battery to the circuit and measure the current again.

6. Repeat until you have four batteries and you have completed your table.
7. Draw a graph of voltage versus current.

**Results:**

1. Does your experimental results verify Ohm's Law? Explain.
2. How would you go about finding the resistance of an unknown resistor using only a power supply, a voltmeter and a known resistor $R_0$?

---

---

**Activity :: Activity : Ohm's Law**

If you do not have access to the equipment necessary for the Ohm's Law experiment, you can do this activity.

| Voltage, $V$ (V) | Current, $I$ (A) |
|---|---|
| 3,0 | 0,4 |
| 6,0 | 0,8 |
| 9,0 | 1,2 |
| 12,0 | 1,6 |

1. Plot a graph of voltage (on the $y$-axis) and current (on the $x$-axis).

**Conclusions:**

1. What type of graph do you obtain (straight line, parabola, other curve)
2. Calculate the gradient of the graph.
3. Does your experimental results verify Ohm's Law? Explain.
4. How would you go about finding the resistance of an unknown resistor using only a power supply, a voltmeter and a known resistor $R_0$?

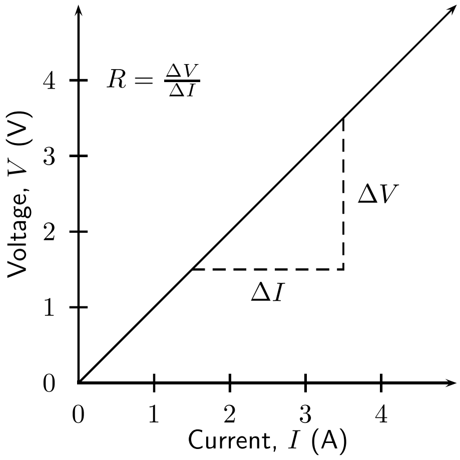
---

An important relationship between the current, voltage and resistance in a circuit was discovered by Georg Simon Ohm and is called **Ohm's Law**.

**Definition: Ohm's Law**
The amount of electric current through a metal conductor, at a constant temperature, in a circuit is proportional to the voltage across the conductor. Mathematically, Ohm's Law is written:

$$V = R \cdot I.$$

Ohm's Law tells us that if a conductor is at a constant temperature, the voltage across the ends of the conductor is proportional to the current. This means that if we plot voltage on the $y$-axis of a graph and current on the $x$-axis of the graph, we will get a straight-line. The gradient of the straight-line graph is then the resistance of the conductor.



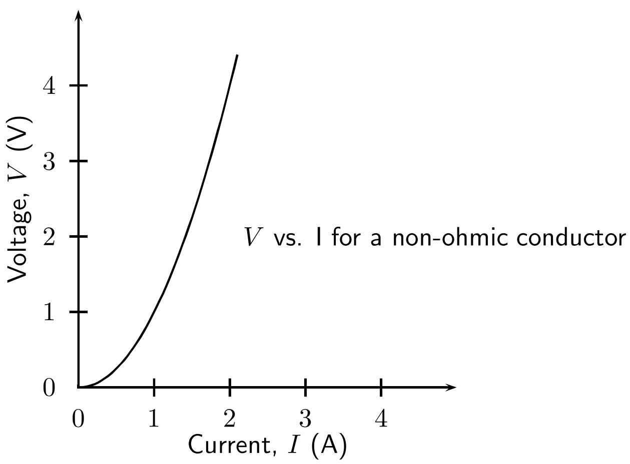

### 19.2.2 Ohmic and non-ohmic conductors

As you have seen, there is a mention of *constant temperature* when we talk about Ohm's Law. This is because the resistance of some conductors change as their temperature changes. These types of conductors are called *non-ohmic* conductors, because they do not obey Ohm's Law. As can be expected, the conductors that obey Ohm's Law are called *ohmic* conductors. A light bulb is a common example of a non-ohmic conductor. Nichrome wire is an ohmic conductor.

In a light bulb, the resistance of the filament wire will increase dramatically as it warms from room temperature to operating temperature. If we increase the supply voltage in a real lamp circuit, the resulting increase in current causes the filament to increase in temperature, which increases its resistance. This effectively limits the increase in current. In this case, voltage and current do not obey Ohm's Law.

The phenomenon of resistance changing with variations in temperature is one shared by almost all metals, of which most wires are made. For most applications, these changes in resistance are small enough to be ignored. In the application of metal lamp filaments, which increase a lot in temperature (up to about 1000°C, and starting from room temperature) the change is quite large.

In general non-ohmic conductors have plots of voltage against current that are curved, indicating that the resistance is not constant over all values of voltage and current.

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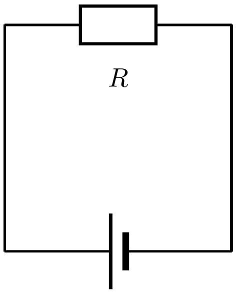
**Activity :: Experiment : Ohmic and non-ohmic conductors**
Repeat the experiment as decribed in the previous section. In this case use a light bulb as a resistor. Compare your results to the ohmic resistor.

### 19.2.3 Using Ohm's Law

We are now ready to see how Ohm's Law is used to analyse circuits.

Consider the circuit with an ohmic resistor, $R$. If the resistor has a resistance of 5 $\Omega$ and voltage across the resistor is 5V, then we can use Ohm's law to calculate the current flowing through the resistor.




Ohm's law is:

$$V = R \cdot I$$

which can be rearranged to:

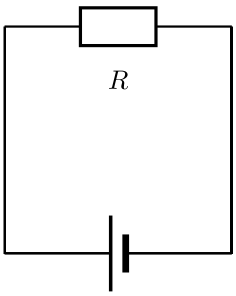
$$I = \frac{V}{R}$$

The current flowing in the resistor is:

$$\begin{aligned} I &= \frac{V}{R} \\ &= \frac{5\ \mathrm{V}}{5\ \Omega} \\ &= 1\ \mathrm{A} \end{aligned}$$

**Worked Example 124: Ohm's Law**

**Question:**




The resistance of the above resistor is 10 $\Omega$ and the current going through the resistor is 4 A. What is the potential difference (voltage) across the resistor?

**Answer**

**Step 1 : Determine how to approach the problem**
It is an Ohm's Law problem. So we use the equation:

$$V = R \cdot I$$

**Step 2 : Solve the problem**

$$\begin{aligned} V &= R \cdot I \\ &= (10)(4) \\ &= 40\ V \end{aligned}$$

**Step 3 : Write the final answer**
The voltage across the resistor is 40 V.

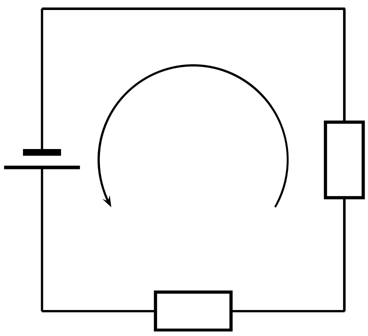
?

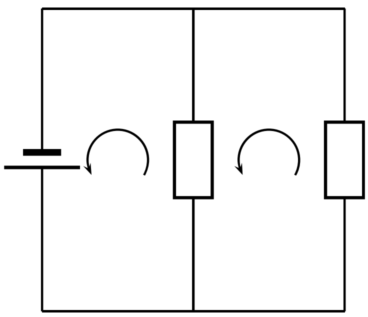
**Exercise: Ohm's Law**

1. Calculate the resistance of a resistor that has a potential difference of 8 V across it when a current of 2 A flows through it.
2. What current will flow through a resistor of 6 Ω when there is a potential difference of 18 V across its ends?
3. What is the voltage acroos a 10 Ω resistor when a current of 1,5 A flows though it?

## 19.3 Resistance

In Grade 10, you learnt about resistors and were introduced to circuits where resistors were connected in series and circuits where resistors were connected in parallel. In a series circuit there is one path for the current to flow through. In a parallel circuit there are multiple paths for the current to flow through.


series circuit
one current path

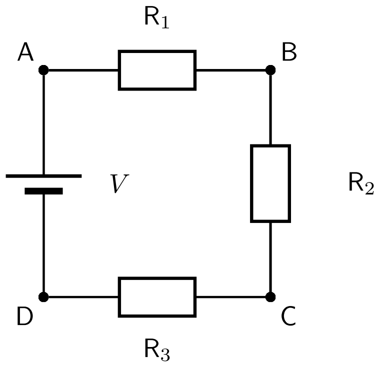

parallel circuit
multiple current paths

### 19.3.1 Equivalent resistance

When there is more than one resistor in a circuit, we are usually able to replace all resistors with a single resistor whose effect is the same as all the resistors put together. The resistance of the single resistor is known as *equivalent resistance*. We are able to calculate equivalent resistance for resistors connected in series and parallel.

**Equivalent Series Resistance**

Consider a circuit consisting of three resistors and a single battery connected in series.




The first principle to understand about series circuits is that the amount of current is the same through any component in the circuit. This is because there is only one path for electrons to flow in a series circuit. From the way that the battery is connected, we can tell which direction the current will flow. We know that charge flows from positive to negative, by convention. Current in this circuit will flow in a clockwise direction, from point A to B to C to D and back to A.

So, how do we use this knowledge to calculate the value of a single resistor that can replace the three resistors in the circuit and still have the same current?

We know that in a series circuit the current has to be the same in all components. So we can write:

$$I = I_1 = I_2 = I_3$$

We also know that total voltage of the circuit has to be equal to the sum of the voltages over all three resistors. So we can write:

$$V = V_1 + V_2 + V_3$$

Finally, we know that Ohm's Law has to apply for each resistor individually, which gives us:

$$\begin{aligned} V_1 &= I_1 \cdot R_1 \\ V_2 &= I_2 \cdot R_2 \\ V_3 &= I_3 \cdot R_3 \end{aligned}$$

Therefore:

$$V = I_1 \cdot R_1 + I_2 \cdot R_2 + I_3 \cdot R_3$$

However, because

$$I = I_1 = I_2 = I_3$$

, we can further simplify this to:

$$\begin{aligned} V &= I \cdot R_1 + I \cdot R_2 + I \cdot R_3 \\ &= I(R_1 + R_2 + R_3) \end{aligned}$$

Further, we can write an Ohm's Law relation for the entire circuit:

$$V = I \cdot R$$

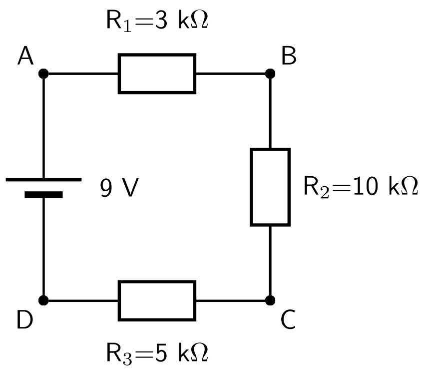
Therefore:

$$\begin{aligned} V &= I(R_1 + R_2 + R_3) \\ I \cdot R &= I(R_1 + R_2 + R_3) \\ \therefore \quad R &= R_1 + R_2 + R_3 \end{aligned}$$

**Definition: Equivalent resistance in a series circuit, $R_s$**
For $n$ resistors in series the equivalent resistance is:

$$R_s = R_1 + R_2 + R_3 + \cdots + R_n$$

Let us apply this to the following circuit.




The resistors are in series, therefore:

$$\begin{aligned} R_s &= R_1 + R_2 + R_3 \\ &= 3\,\Omega + 10\,\Omega + 5\,\Omega \\ &= 18\,\Omega \end{aligned}$$

**Worked Example 125: Equivalent series resistance I**

**Question:** Two 10 k$\Omega$ resistors are connected in series. Calculate the equivalent resistance.

**Answer**

**Step 1 : Determine how to approach the problem**

Since the resistors are in series we can use:

$$R_s = R_1 + R_2$$

**Step 2 : Solve the problem**

$$\begin{aligned} R_s &= R_1 + R_2 \\ &= 10\,\text{k}\,\Omega + 10\,\text{k}\,\Omega \\ &= 20\,\text{k}\,\Omega \end{aligned}$$

**Step 3 : Write the final answer**

The equivalent resistance of two 10 k$\Omega$ resistors connected in series is 20 k$\Omega$.

**Worked Example 126: Equivalent series resistance II**

**Question:** Two resistors are connected in series. The equivalent resistance is 100 Ω. If one resistor is 10 Ω, calculate the value of the second resistor.

**Answer**

**Step 1 : Determine how to approach the problem**

Since the resistors are in series we can use:

$$R_s = R_1 + R_2$$

We are given the value of $R_s$ and $R_1$.

**Step 2 : Solve the problem**

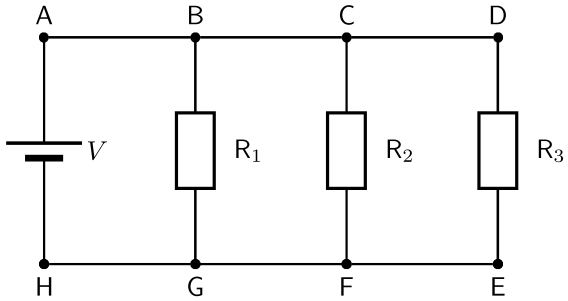
$$\begin{aligned} R_s &= R_1 + R_2 \\ \therefore \quad R_2 &= R_s - R_1 \\ &= 100\,\Omega - 10\,\Omega \\ &= 90\,\Omega \end{aligned}$$

**Step 3 : Write the final answer**

The second resistor has a resistance of 90Ω.

### Equivalent parallel resistance

Consider a circuit consisting of a single battery and three resistors that are connected in parallel.




The first principle to understand about parallel circuits is that the voltage is equal across all components in the circuit. This is because there are only two sets of electrically common points in a parallel circuit, and voltage measured between sets of common points must always be the same at any given time. So, for the circuit shown, the following is true:

$$V = V_1 = V_2 = V_3$$

The second principle for a parallel circuit is that all the currents through each resistor must add up to the total current in the circuit.

$$I = I_1 + I_2 + I_3$$

Also, from applying Ohm's Law to the entire circuit, we can write:

$$V = \frac{I}{R_p}$$

where $R_p$ is the equivalent resistance in this parallel arrangement.

We are now ready to apply Ohm's Law to each resistor, to get:

$$\begin{aligned} V_1 &= R_1 \cdot I_1 \\ V_2 &= R_2 \cdot I_2 \\ V_3 &= R_3 \cdot I_3 \end{aligned}$$

This can be also written as:

$$\begin{aligned} I_1 &= \frac{V_1}{R_1} \\ I_2 &= \frac{V_2}{R_2} \\ I_3 &= \frac{V_3}{R_3} \end{aligned}$$

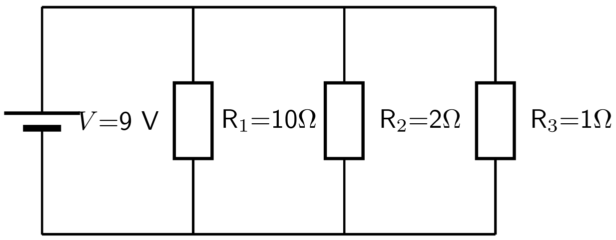
Now we have:

$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ \frac{V}{R_p} &= \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \\ &= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \\ \text{because} \quad & V = V_1 = V_2 = V_3 \\ &= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \\ \therefore \quad \frac{1}{R_p} &= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \end{aligned}$$

**Definition: Equivalent resistance in a parallel circuit, $R_p$**
For $n$ resistors in parallel, the equivalent resistance is:

$$\frac{1}{R_p} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}\right)$$

Let us apply this formula to the following circuit.



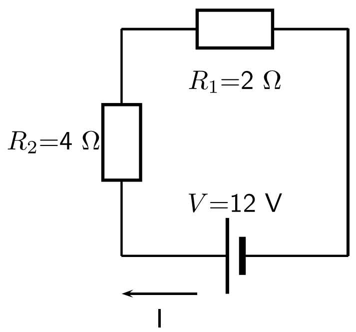

$$\begin{aligned} \frac{1}{R_p} &= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \\ &= \left(\frac{1}{10\,\Omega} + \frac{1}{2\,\Omega} + \frac{1}{1\,\Omega}\right) \\ &= \left(\frac{1+5+10}{10}\right) \\ &= \left(\frac{16}{10}\right) \\ \therefore \quad R_p &= \frac{10}{16}\,\Omega \end{aligned}$$

### 19.3.2 Use of Ohm's Law in series and parallel Circuits

**Worked Example 127: Ohm's Law**

**Question:** Calculate the current $(I)$ in this circuit if the resistors are both ohmic in nature.

**Answer**




**Step 1 : Determine what is required**

We are required to calculate the current flowing in the circuit.

**Step 2 : Determine how to approach the problem**

Since the resistors are Ohmic in nature, we can use Ohm's Law. There are however two resistors in the circuit and we need to find the total resistance.

**Step 3 : Find total resistance in circuit**

Since the resistors are connected in series, the total resistance $R$ is:

$$R = R_1 + R_2$$

Therefore,

$$R = 2 + 4 = 6\ \Omega$$

**Step 4 : Apply Ohm's Law**

$$\begin{aligned} V &= R \cdot I \\ \therefore \quad I &= \frac{V}{R} \\ &= \frac{12}{6} \\ &= 2\ \text{A} \end{aligned}$$

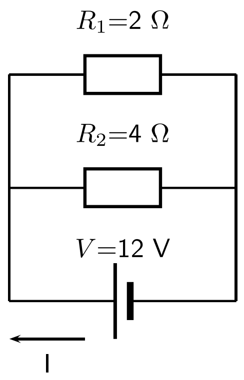
**Step 5 : Write the final answer**

A 2 A current is flowing in the circuit.

**Worked Example 128: Ohm's Law I**

**Question:** Calculate the current $(I)$ in this circuit if the resistors are both ohmic in nature.

**Answer**




**Step 1 : Determine what is required**
We are required to calculate the current flowing in the circuit.
**Step 2 : Determine how to approach the problem**
Since the resistors are Ohmic in nature, we can use Ohm's Law. There are however two resistors in the circuit and we need to find the total resistance.
**Step 3 : Find total resistance in circuit**
Since the resistors are connected in parallel, the total resistance $R$ is:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

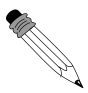
Therefore,

$$\begin{aligned} \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2} \\ &= \frac{1}{2} + \frac{1}{4} \\ &= \frac{2+1}{4} \\ &= \frac{3}{4} \\ Therefore, R &= \frac{4}{3}\ \Omega \end{aligned}$$

**Step 4 : Apply Ohm's Law**

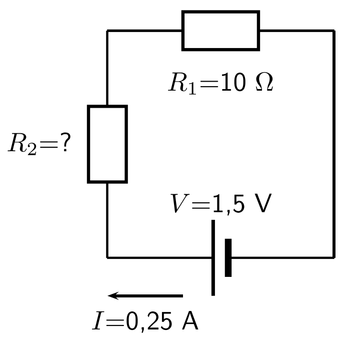
$$\begin{aligned} V &= R \cdot I \\ \therefore \quad I &= \frac{V}{R} \\ &= \frac{12}{\frac{4}{3}} \\ &= 9\ \mathrm{A} \end{aligned}$$

**Step 5 : Write the final answer**
A 9 A current is flowing in the circuit.



**Worked Example 129: Ohm's Law II**

**Question:** Two ohmic resistors ($R_1$ and $R_2$) are connected in series with a battery. Find the resistance of $R_2$, given that the current flowing through $R_1$ and $R_2$ is 0,25 A and that the voltage across the battery is 1,5 V. $R_1$=1 Ω.
**Answer**
**Step 6 : Draw the circuit and fill in all known values.**




**Step 7 : Determine how to approach the problem.**

We can use Ohm's Law to find the total resistance $R$ in the circuit, and then calculate the unknown resistance using:

$$R = R_1 + R_2$$

in a series circuit.

**Step 8 : Find the total resistance**

$$\begin{aligned} V &= R \cdot I \\ \therefore \quad R &= \frac{V}{I} \\ &= \frac{1,5}{0,25} \\ &= 6\,\Omega \end{aligned}$$

**Step 9 : Find the unknown resistance**

We know that:

$$R = 6\,\Omega$$

and that

$$R_1 = 1\,\Omega$$

Since

$$R = R_1 + R_2$$

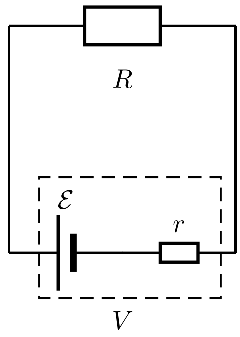
$$R_2 = R - R_1$$

Therefore,

$$R_2 = 5\,\Omega$$

### 19.3.3 Batteries and internal resistance

Real batteries are made from materials which have resistance. This means that real batteries are not just sources of potential difference (voltage), but they also possess internal resistances. If the pure voltage source is referred to as the emf, $\mathcal{E}$, then a real battery can be represented as an emf connected in series with a resistor $r$. The internal resistance of the battery is represented by the symbol $r$.




**Definition: Load**
The external resistance in the circuit is referred to as the load.

Suppose that the battery (or cell) with emf $\mathcal{E}$ and internal resistance $r$ supplies a current $I$ through an external load resistor $R$. Then the voltage drop across the load resistor is that supplied by the battery:

$$V = I \cdot R$$

Similarly, from Ohm's Law, the voltage drop across the internal resistance is:

$$V_r = I \cdot r$$

The voltage $V$ of the battery is related to its emf $\mathcal{E}$ and internal resistance $r$ by:

$$\begin{aligned} \mathcal{E} &= V + Ir; or \\ V &= \mathcal{E} - Ir \end{aligned}$$

The emf of a battery is essentially constant because it only depends on the chemical reaction (that converts chemical energy into electrical energy) going on inside the battery. Therefore, we can see that the voltage across the terminals of the battery is dependent on the current drawn by the load. The higher the current, the lower the voltage across the terminals, because the emf is constant. By the same reasoning, the voltage only equals the emf when the current is very small.

The maximum current that can be drawn from a battery is limited by a critical value $I_c$. At a current of $I_c$, $V$=0 V. Then, the equation becomes:

$$\begin{aligned} 0 &= \mathcal{E} - I_c r \\ I_c r &= \mathcal{E} \\ I_c &= \frac{\mathcal{E}}{r} \end{aligned}$$

The maximum current that can be drawn from a battery is less than $\frac{\mathcal{E}}{r}$.

**Worked Example 130: Internal resistance**

**Question:** What is the internal resistance of a battery if its emf is 12 V and the voltage drop across its terminals is 10 V when a current of 4 A flows in the circuit when it is connected across a load?

**Answer**

**Step 1 : Determine how to approach the problem**

It is an internal resistance problem. So we use the equation:

$$\mathcal{E} = V + Ir$$

**Step 2 : Solve the problem**

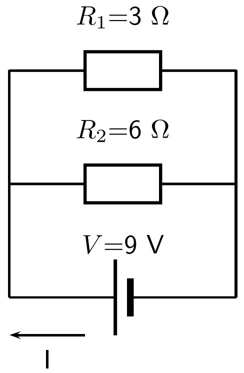
$$\begin{aligned} \mathcal{E} &= V + Ir \\ 12 &= 10 + 4(r) \\ &= 0.5 \end{aligned}$$

**Step 3 : Write the final answer**
The internal resistance of the resistor is 0.5 Ω.

?

**Exercise: Resistance**

1. Calculate the equivalent resistance of:
   A three 2 Ω resistors in series;
   B two 4 Ωresistors in parallel;
   C a 4 Ω resistor in series with a 8 Ω resistor;
   D a 6 Ω resistor in series with two resistors (4 Ω and 2Ω ) in parallel.
2. Calculate the current in this circuit if both resistors are ohmic.




3. Two ohmic resistors are connected in series. The resistance of the one resistor is 4 Ω̇What is the resistance of the other resistor if a current of 0,5 A flows through the resistors when they are connected to a voltage supply of 6 V.
4. Describe what is meant by the *internal resistance* of a real battery.
5. Explain why there is a difference between the emf and terminal voltage of a battery if the load (external resistance in the circuit) is comparable in size to the battery's internal resistance
6. What is the internal resistance of a battery if its emf is 6 V and the voltage drop across its terminals is 5,8 V when a current of 0,5 A flows in the circuit when it is connected across a load?

## 19.4 Series and parallel networks of resistors

Now that you know how to handle simple series and parallel circuits, you are ready to tackle problems like this:

It is relatively easy to work out these kind of circuits because you use everything you have already learnt about series and parallel circuits. The only difference is that you do it in stages.

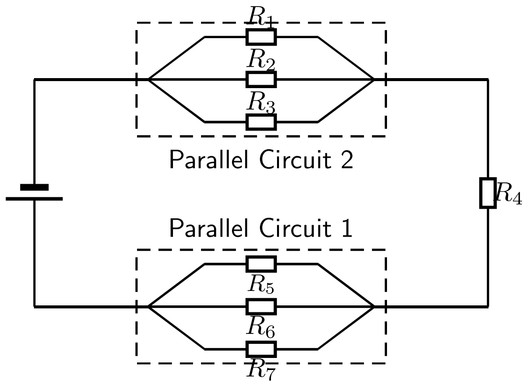
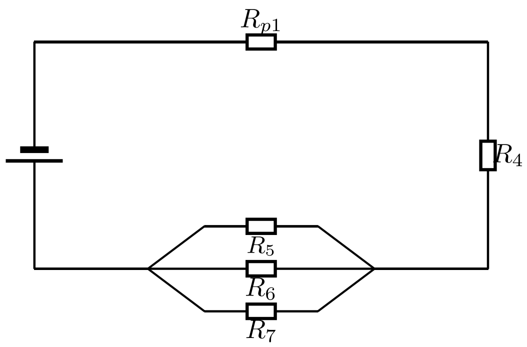



Figure 19.1: An example of a series-parallel network. The dashed boxes indicate parallel sections of the circuit.

In Figure 19.1, the circuit consists of 2 parallel portions that are then in series with 1 resistor. So, in order to work out the equivalent resistance, you start by reducing the parallel portions to a single resistor and then add up all the resistances in series. If all the resistors in Figure 19.1 had resistances of 10 Ω, we can calculate the equivalent resistance of the entire circuit.

We start by reducing *Parallel Circuit 1* to a single resistor.

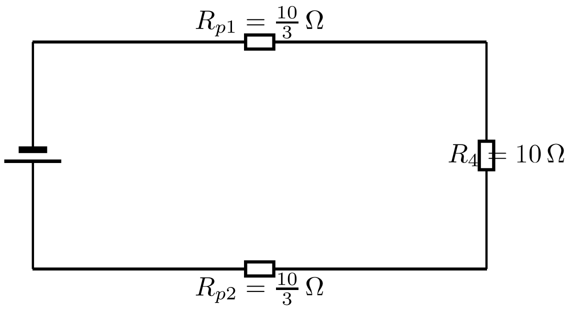



The value of $R_{p1}$ is:

$$\begin{aligned}
\frac{1}{R_{p1}} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \\
R_{p1} &= \left(\frac{1}{10} + \frac{1}{10} + \frac{1}{10}\right)^{-1} \\
&= \left(\frac{1+1+1}{10}\right)^{-1} \\
&= \left(\frac{3}{10}\right)^{-1} \\
&= \frac{10}{3}\,\Omega
\end{aligned}$$

We can similarly replace *Parallel Circuit 2* with $R_{p2}$ which has a value given by:

$$\begin{aligned}\frac{1}{R_{p2}} &= \frac{1}{R_5}+\frac{1}{R_6}+\frac{1}{R_7}\\ R_{p2} &= \left(\frac{1}{10}+\frac{1}{10}+\frac{1}{10}\right)^{-1}\\ &= \left(\frac{1+1+1}{10}\right)^{-1}\\ &= \left(\frac{3}{10}\right)^{-1}\\ &= \frac{10}{3}\,\Omega\end{aligned}$$




This is now a simple series circuit and the equivalent resistance is:

$$\begin{aligned}R &= R_{p1}+R_4+R_{p2}\\ &= \frac{10}{3}+10+\frac{10}{3}\\ &= \frac{100+30+100}{30}\\ &= \frac{230}{30}\\ &= 7\frac{2}{3}\,\Omega\end{aligned}$$

The equivalent resistance of the circuit in Figure 19.1 is $7\frac{2}{3}\,\Omega$.

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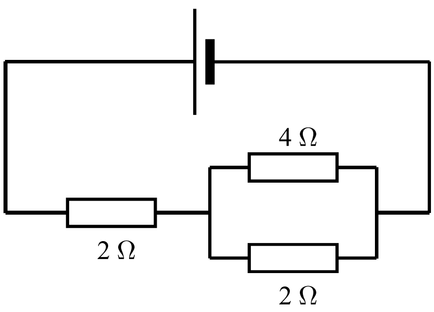
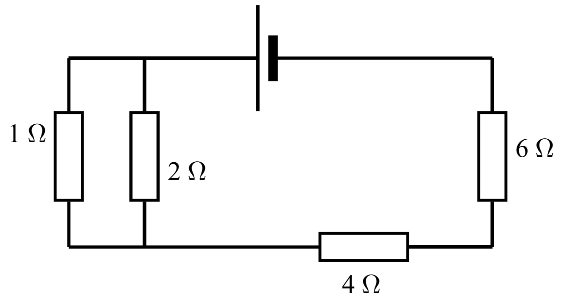
**Exercise: Series and parallel networks**

Determine the equivalent resistance of the following circuits:

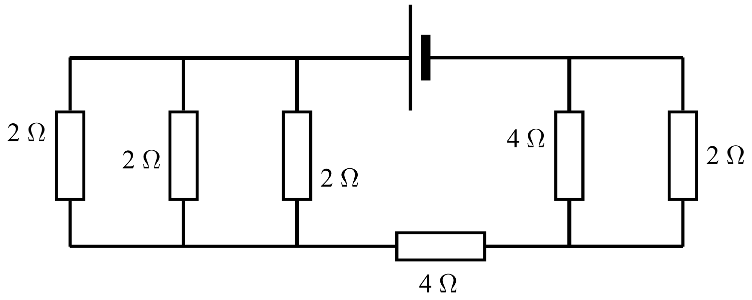
1. Hello



2.



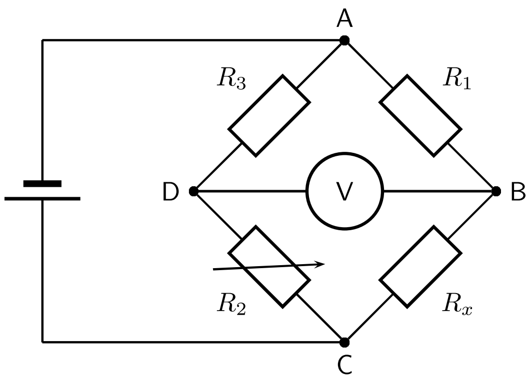

3.




## 19.5 Wheatstone bridge

Another method of finding an unknown resistance is to use a *Wheatstone bridge*. A Wheatstone bridge is a measuring instrument that is used to measure an unknown electrical resistance by balancing two legs of a bridge circuit, one leg of which includes the unknown component. Its operation is similar to the original potentiometer except that in potentiometer circuits the meter used is a sensitive galvanometer.

The Wheatstone bridge was invented by Samuel Hunter Christie in 1833 and improved and popularized by Sir Charles Wheatstone in 1843.

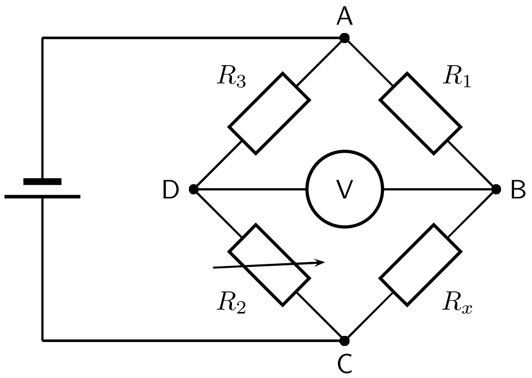



Circuit for Wheatstone bridge

In the circuit of the Wheatstone bridge, $R_x$ is the unknown resistance. $R_1$, $R_2$ and $R_3$ are resistors of known resistance and the resistance of $R_2$ is adjustable. If the ratio of $R_2$:$R_1$ is equal to the ratio of $R_x$:$R_3$, then the voltage between the two midpoints will be zero and no current will flow between the midpoints. In order to determine the unknown resistance, $R_2$ is varied until this condition is reached. That is when the voltmeter reads 0 V.

**Worked Example 131: Wheatstone bridge**

**Question:**
**Answer**
What is the resistance of the unknown resistor $R_x$ in the diagram below if $R_1$=4Ω $R_2$=8Ω and $R_3$=6Ω.




Circuit for Wheatstone bridge

**Step 1 : Determine how to approach the problem**
The arrangement is a Wheatstone bridge. So we use the equation:

$$R_x : R_3 = R_2 : R_1$$

**Step 2 : Solve the problem**

$$\begin{aligned} R_x : R_3 &= R_2 : R_1 \\ R_x : 6 &= 8 : 4 \\ R_x &= 12\ \Omega \end{aligned}$$

**Step 3 : Write the final answer**
The resistance of the unknown resistor is 12 Ω.

*Extension: Power in electric circuits*
In addition to voltage and current, there is another measure of free electron activity in a circuit: *power*. Power is a measure of how rapidly a standard amount of *work* is done. In electric circuits, power is a function of both voltage and current:

**Definition: Electrical Power**
Electrical power is calculated as:

$$P = I \cdot V$$

Power ($P$) is exactly equal to current ($I$) multiplied by voltage ($V$) and there is no extra constant of proportionality. The unit of measurement for power is the *Watt* (abbreviated W).

**Interesting Fact**

It was James Prescott Joule, not Georg Simon Ohm, who first discovered the mathematical relationship between power dissipation and current through a resistance. This discovery, published in 1841, followed the form of the equation:

$$P = I^2R$$

and is properly known as Joule's Law. However, these power equations are so commonly associated with the Ohm's Law equations relating voltage, current, and resistance that they are frequently credited to Ohm.

---

**Activity :: Investigation : Equivalence**
Use Ohm's Law to show that:

$$P = VI$$

is identical to

$$P = I^2R$$

and

$$P = \frac{V^2}{R}$$

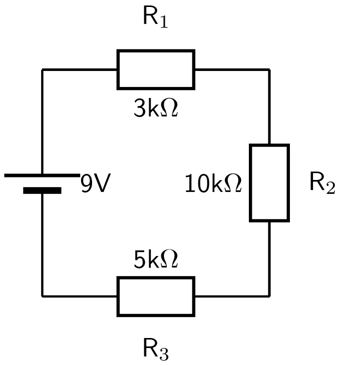
---

## 19.6 Summary

1. Ohm's Law states that the amount of current through a conductor, at constant temperature, is proportional to the voltage across the resistor. Mathematically we write $V = R/cdotI$
2. Conductors that obey Ohm's Law are called ohmic conductors; those who do not are called non-ohmic conductors.
3. We use Ohm's Law to calculate the resistance of a resistor. $(R = \frac{V}{I}$
4. The equivalent resistance of resistors in series ($R_s$) can be calculated as follows: $R_s = R_1 + R_2 + R_3 + ... + R_n$
5. The equivalent resistance of resistors in parallel ($R_p$) can be calculated as follows: $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$
6. Real batteries have an internal resistance.
7. Wheatstone bridges can be used to accurately determine the resistance of an unknown resistor.

## 19.7 End of chapter exercise

1. Calculate the current in the following circuit and then use the current to calculate the voltage drops across each resistor.




2. Explain why a voltmeter is placed in parallel with a resistor.

3. Explain why an ammeter is placed in series with a resistor.

4. [IEB 2001/11 HG1] - **Emf**

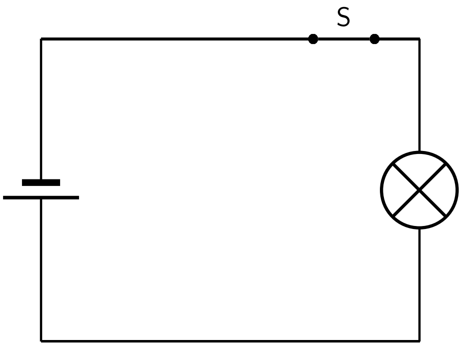
   A Explain the meaning of each of these two statements:

      i. "The current through the battery is 50 mA."

      ii. "The emf of the battery is 6 V."

   B A battery tester measures the current supplied when the battery is connected to a resistor of 100 Ω. If the current is less than 50 mA, the battery is "flat" (it needs to be replaced). Calculate the maximum internal resistance of a 6 V battery that will pass the test.

5. [IEB 2005/11 HG] The electric circuit of a torch consists of a cell, a switch and a small light bulb.



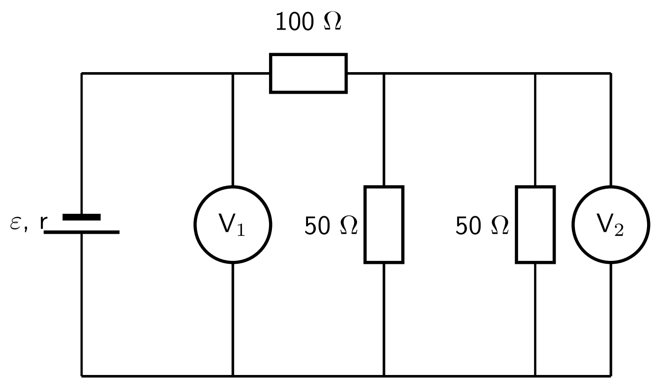

The electric torch is designed to use a D-type cell, but the only cell that is available for use is an AA-type cell. The specifications of these two types of cells are shown in the table below:

| Cell | emf | Appliance for which it is designed | Current drawn from cell when connected to the appliance for which it is designed |
|---|---|---|---|
| D | 1,5 V | torch | 300 mA |
| AA | 1,5 V | TV remote control | 30 mA |

What is likely to happen and why does it happen when the AA-type cell replaces the D-type cell in the electric torch circuit?

| | What happens | Why it happens |
|---|---|---|
| (a) | the bulb is dimmer | the AA-type cell has greater internal resistance |
| (b) | the bulb is dimmer | the AA-type cell has less internal resistance |
| (c) | the brightness of the bulb is the same | the AA-type cell has the same internal resistance |
| (d) | the bulb is brighter | the AA-type cell has less internal resistance |

6. [IEB 2005/11 HG1] A battery of emf $\varepsilon$ and internal resistance r = 25 Ω is connected to this arrangement of resistors.




The resistances of voltmeters $V_1$ and $V_2$ are so high that they do not affect the current in the circuit.

A Explain what is meant by "the emf of a battery".
The power dissipated in the 100 Ω resistor is 0,81 W.

B Calculate the current in the 100 Ω resistor.

C Calculate the reading on voltmeter $V_2$.

D Calculate the reading on voltmeter $V_1$.

E Calculate the emf of the battery.

7. [SC 2003/11] A kettle is marked 240 V; 1 500 W.

A Calculate the resistance of the kettle when operating according to the above specifications.

B If the kettle takes 3 minutes to boil some water, calculate the amount of electrical energy transferred to the kettle.

8. [IEB 2001/11 HG1] - **Electric Eels**
Electric eels have a series of cells from head to tail. When the cells are activated by a nerve impulse, a potential difference is created from head to tail. A healthy electric eel can produce a potential difference of 600 V.

A What is meant by "a potential difference of 600 V"?

B How much energy is transferred when an electron is moved through a potential difference of 600 V?