Chapter 4

Gravity and Mechanical Energy - Grade 10

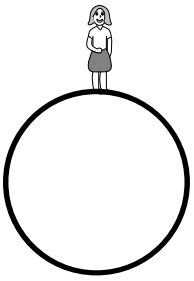
4.1 Weight

Weight is the gravitational force that the Earth exerts on any object. The weight of an objects gives you an indication of how strongly the Earth attracts that body towards its centre. Weight is calculated as follows:

$$\text{Weight} = \text{mg}$$

where m = mass of the object (in kg)
and g = the acceleration due to gravity (9,8 m·s^{-2})

For example, what is Sarah's weight if her mass is 50 kg. Sarah's weight is calculated according to:



$$\begin{aligned} \text{Weight} &= mg \\ &= (50\ \text{kg})(9{,}8\,\text{m}\cdot\text{s}^{-2}) \\ &= 490\ \text{kg}\cdot\ \text{m}\cdot\text{s}^{-2} \\ &= 490\ \text{N} \end{aligned}$$

Important: Weight is sometimes abbreviated as F_g which refers to the force of gravity. Do not use the abbreviation 'W' for weight as it refers to 'Work'.

Now, we have said that the value of g is approximately 9,8 m·s^{-2}on the surface of the Earth. The actual value varies slightly over the surface of the Earth. Each planet in our Solar System has its own value for g. These values are listed as multiples of g on Earth in Table 4.1

Worked Example 15: Determining mass and weight on other planets

Question: Sarah's mass on Earth is 50 kg. What is her mass and weight on Mars?
Answer
Step 1 : Determine what information is given and what is asked
m (on Earth) = 50 kg
m (on Mars) = ?
Weight (on Mars) = ?

Planet	Gravitational Acceleration (multiples of g on Earth)
Mercury	0.376
Venus	0.903
Earth	1
Mars	0.38
Jupiter	2.34
Saturn	1.16
Uranus	1.15
Neptune	1.19
Pluto	0.066

Table 4.1: A list of the gravitational accelerations at the surfaces of each of the planets in our solar system. Values are listed as multiples of g on Earth. **Note**: The "surface" is taken to mean the cloud tops of the gas giants (Jupiter, Saturn, Uranus and Neptune).

Step 2 : Calculate her mass on Mars
Sarah's mass does not change because she is still made up of the same amount of matter. Her mass on Mars is therefore 50 kg.

Step 3 : Calculate her weight on Mars

$$\begin{aligned} \text{Sarah's weight} &= 50 \times 0{,}38 \times 9{,}8 \\ &= 186{,}2 \text{ N} \end{aligned}$$

4.1.1 Differences between Mass and Weight

Mass is measured in kilograms (kg) and is the amount of matter in an object. An object's mass does not change unless matter is added or removed from the object.

The differences between mass and weight can be summarised in the following table:

Mass	**Weight**
1. is a measure of how many molecules there are in an object.	1. is the force with which the Earth attracts an object.
2. is measured in kilograms.	2. is measured in newtons
3. is the same on any planet.	3. is different on different planets.
4. is a scalar.	4. is a vector.

Exercise: Weight

1. A bag of sugar has a mass of 1 kg. How much does it weigh:
   (a) on Earth?
   (b) on Jupiter?
   (c) on Pluto?
2. Neil Armstrong was the first man to walk on the surface of the Moon. The gravitational acceleration on the Moon is $\frac{1}{6}$ of the gravitational acceleration on Earth, and there is no gravitational acceleration in outer space. If Neil's mass was 90 kg, what was his weight:
   (a) on Earth?

(b) on the Moon?
(c) in outer space?

3. A monkey has a mass of 15 kg on Earth. The monkey travels to Mars. What is his mass and weight on Mars?
4. Determine your mass by using a bathroom scale and calculate your weight for each planet in the Solar System, using the values given in Table 4.1

4.2 Acceleration due to Gravity

4.2.1 Gravitational Fields

A *field* is a region of space in which a mass experiences a force. Therefore, a *gravitational* field is a region of space in which a mass experiences a gravitational force.

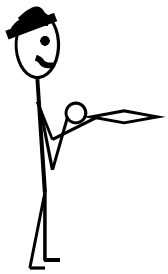
4.2.2 Free fall

Important: Free fall is motion in the Earth's gravitational field when no other forces act on the object.

Free fall is the term used to describe a special kind of motion in the Earth's gravitational field. Free fall is motion in the Earth's gravitational field when no other forces act on the object. It is basically an ideal situation, since in reality, there is always some air friction which slows down the motion.

Activity :: Experiment : Acceleration due to Gravity
Aim: Investigating the acceleration of two different objects during free fall.
Apparatus: Tennis ball and a sheet of A4 paper.
Method:

1. Hold the tennis ball and sheet of paper (horizontally) the same distance from the ground. Which one would strike the ground first if both were dropped?



2. Drop both objects and observe. Explain your observations.
3. Now crumple the paper into a ball, more or less the same size as the tennis ball. Drop the paper and tennis ball again and observe. Explain your observations.
4. Why do you think the two situations are different?
5. Compare the value for the acceleration due to gravity of the tennis ball to the crumpled piece of paper.
6. Predict what will happen if an iron ball and a tennis ball of the same size are dropped from the same height. What will the values for their acceleration due to gravity be?

If a metal ball and tennis ball (of the same size) were dropped from the same height, both would reach the ground at the same time. It does not matter that the one ball is heavier than the other. The acceleration of an object due to gravity is independent of the mass of the object. It does not matter what the mass of the object is.

The shape of the object, however, is important. The sheet of paper took much longer to reach the ground than the tennis ball. This is because the effect of air friction on the paper was much greater than the air friction on the tennis ball.

If we lived in a world where there was no air resistance, the A4 sheet of paper and the tennis ball would reach the ground at the same time. This happens in outer space or in a vaccuum.

Galileo Galilei, an Italian scientist, studied the motion of objects. The following case study will tell you more about one of his investigations.

Activity :: Case Study : Galileo Galilei

In the late sixteenth century, it was generally believed that heavier objects would fall faster than lighter objects. The Italian scientist Galileo Galilei thought differently. Galileo hypothesized that two objects would fall at the same rate regardless of their mass. Legend has it that in 1590, Galileo planned out an experiment. He climbed to the top of the Leaning Tower of Pisa and dropped several large objects to test his theory. He wanted to show that two different objects fall at the same rate (as long as we ignore air resistance). Galileo's experiment proved his hypothesis correct; the acceleration of a falling object is independent of the object's mass.

A few decades after Galileo, Sir Isaac Newton would show that acceleration depends upon both force and mass. While there is greater force acting on a larger object, this force is canceled out by the object's greater mass. Thus two objects will fall (actually they are pulled) to the earth at exactly the same rate.

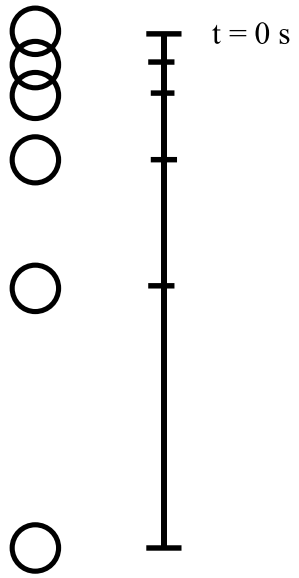
Questions: Read the case study above and answer the following questions.

1. Divide into pairs and explain Galileo's experiment to your friend.
2. Write down an aim and a hypothesis for Galileo's experiment.
3. Write down the result and conclusion for Galileo's experiment.

Activity :: Research Project : Experimental Design

Design an experiment similar to the one done by Galileo to prove that the acceleration due to gravity of an object is independent of the object's mass. The investigation must be such that you can perform it at home or at school. Bring your apparatus to school and perform the experiment. Write it up and hand it in for assessment.

Activity :: Case Study : Determining the acceleration due to gravity 1
Study the set of photographs alongside and answer the following questions:



1. Determine the time between each picture if the frequency of the exposures were 10 Hz.
2. Determine the distance between each picture.
3. Calculate the velocity of the ball between pictures 1 and 3.
$$v = \frac{x_3 - x_1}{t_3 - t_1}$$
4. Calculate the velocity of the ball between pictures 4 and 6.
5. Calculate the acceleration the ball between pictures 2 and 5.
$$a = \frac{v_5 - v_2}{t_5 - t_2}$$
6. Compare your answer to the value for the acceleration due to gravity (9,8 m·s^{-2}).

The acceleration due to gravity is constant. This means we can use the equations of motion under constant acceleration that we derived in Chapter 3 (on Page 23) to describe the motion of an object in free fall. The equations are repeated here for ease of use.

v_i = initial velocity (m·s^{-1}) at $t = 0$ s
v_f = final velocity (m·s^{-1}) at time t
Δx = displacement (m)
t = time (s)
Δt = time interval (s)
g = acceleration (m·s^{-2})

$$v_f = v_i + gt \tag{4.1}$$
$$\Delta x = \frac{(v_i + v_f)}{2}t \tag{4.2}$$
$$\Delta x = v_i t + \frac{1}{2}gt^2 \tag{4.3}$$
$$v_f^2 = v_i^2 + 2g\Delta x \tag{4.4}$$

Activity :: Experiment : Determining the acceleration due to gravity 2
Work in groups of at least two people.
Aim: To determine the acceleration of an object in freefall.

Apparatus: Large marble, two stopwatches, measuring tape.
Method:

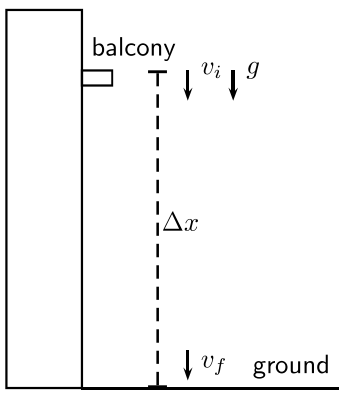
1. Measure the height of a door, from the top of the door to the floor, exactly. Write down the measurement.
2. One person must hold the marble at the top of the door. Drop the marble to the floor at the same time as he/she starts the first stopwatch.
3. The second person watches the floor and starts his stopwatch when the marble hits the floor.
4. The two stopwatches are stopped together and the two times substracted. The difference in time will give the time taken for the marble to fall from the top of the door to the floor.
5. Design a table to show the results of your experiment. Choose appropriate headings and units.
6. Choose an appropriate equation of motion to calculate the acceleration of the marble. Remember that the marble starts from rest and that it's displacement was determined in the first step.
7. Write a conclusion for your investigation.
8. Answer the following questions:
   (a) Why do you think two stopwatches were used in this investigation?
   (b) Compare the value for acceleration obtained in your investigation with the value of acceleration due to gravity (9,8 m·s^{-2}). Explain your answer.

Worked Example 16: A freely falling ball

Question: A ball is dropped from the balcony of a tall building. The balcony is 15 m above the ground. Assuming gravitational acceleration is 9,8 m·s^{-2}, find:

1. the time required for the ball to hit the ground, and
2. the velocity with which it hits the ground.

Answer
Step 1 : Draw a rough sketch of the problem
It always helps to understand the problem if we draw a picture like the one below:



Step 2 : Identify what information is given and what is asked for
We have these quantities:

$$\begin{aligned} \Delta x &= 15\ \mathrm{m} \\ v_i &= 0\ \mathrm{m \cdot s^{-1}} \\ g &= 9{,}8\ \mathrm{m \cdot s^{-2}} \end{aligned}$$

Step 3 : Choose up or down as the positive direction
Since the ball is falling, we choose down as positive. This means that the values for v_i Δx and a will be positive.
Step 4 : Choose the most appropriate equation.
We can use equation 21.3 to find the time: $\Delta x = v_i t + \frac{1}{2}gt^2$
Step 5 : Use the equation to find t.

$$\begin{aligned}
\Delta x &= v_i t + \frac{1}{2}gt^2 \\
15 &= (0)t + \frac{1}{2}(9{,}8)(t)^2 \\
15 &= 4{,}9\ t^2 \\
t^2 &= 3.0612... \\
t &= 1{,}7496... \\
t &= 1{,}75\ s
\end{aligned}$$

Step 6 : Find the final velocity v_f.
Using equation 21.1 to find v_f:

$$\begin{aligned}
v_f &= v_i + gt \\
v_f &= 0 + (9{,}8)(1{,}7496...) \\
v_f &= 17{,}1464...
\end{aligned}$$

Remember to add the direction: v_f = 17,15 m·s^{-1}downwards.

By now you should have seen that free fall motion is just a special case of motion with constant acceleration, and we use the same equations as before. The only difference is that the value for the acceleration, a, is always equal to the value of gravitational acceleration, g. In the equations of motion we can replace a with g.

?

Exercise: Gravitational Acceleration

1. A brick falls from the top of a 5 m high building. Calculate the velocity with which the brick reaches the ground. How long does it take the brick to reach the ground?
2. A stone is dropped from a window. It takes the stone 1,5 seconds to reach the ground. How high above the ground is the window?
3. An apple falls from a tree from a height of 1,8 m. What is the velocity of the apple when it reaches the ground?

4.3 Potential Energy

The potential energy of an object is generally defined as the energy an object has because of its position relative to other objects that it interacts with. There are different kinds of potential energy such as gravitional potential energy, chemical potential energy, electrical potential energy, to name a few. In this section we will be looking at gravitational potential energy.

Definition: Potential energy
Potential energy is the energy an object has due to its position or state.

Gravitational potential energy is the energy of an object due to its position above the surface of the Earth. The symbol PE is used to refer to gravitational potential energy. You will often find that the words potential energy are used where *gravitational* potential energy is meant. We can define potential energy (or gravitational potential energy, if you like) as:

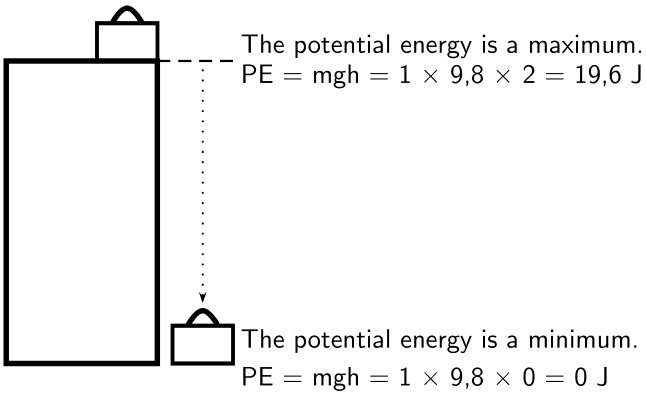
$$PE = mgh \tag{4.5}$$

where PE = potential energy measured in joules (J)
m = mass of the object (measured in kg)
g = gravitational acceleration (9,8 m·s^{-2})
h = perpendicular height from the reference point (measured in m)

A suitcase, with a mass of 1 kg, is placed at the top of a 2 m high cupboard. By lifting the suitcase against the force of gravity, we give the suitcase potential energy. This potential energy can be calculated using equation 4.5.

If the suitcase falls off the cupboard, it will lose its potential energy. Halfway down the cupboard, the suitcase will have lost half its potential energy and will have only 9,8 J left. At the bottom of the cupboard the suitcase will have lost all it's potential energy and it's potential energy will be equal to zero.

Objects have maximum potential energy at a maximum height and will lose their potential energy as they fall.



Worked Example 17: Gravitational potential energy

Question: A brick with a mass of 1 kg is lifted to the top of a 4 m high roof. It slips off the roof and falls to the ground. Calculate the potential energy of the brick at the top of the roof and on the ground once it has fallen.

Answer

Step 1 : Analyse the question to determine what information is provided

- The mass of the brick is $m = 1$ kg
- The height lifted is $h = 4$ m

All quantities are in SI units.

Step 2 : Analyse the question to determine what is being asked

- We are asked to find the gain in potential energy of the brick as it is lifted onto the roof.

- We also need to calculate the potential energy once the brick is on the ground again.

Step 3 : Identify the type of potential energy involved
Since the block is being lifted we are dealing with gravitational potential energy. To work out PE, we need to know the mass of the object and the height lifted. As both of these are given, we just substitute them into the equation for PE.
Step 4 : Substitute and calculate

$$\begin{aligned} PE &= mgh \\ &= (1)(9{,}8)(4) \\ &= 39{,}2\,\text{J} \end{aligned}$$

Exercise: Gravitational Potential Energy

1. Describe the relationship between an object's gravitational potential energy and its:
   (a) mass and
   (b) height above a reference point.
2. A boy, of mass 30 kg, climbs onto the roof of their garage. The roof is 2,5 m from the ground. He now jumps off the roof and lands on the ground.
   (a) How much potential energy has the boy gained by climbing on the roof?
   (b) The boy now jumps down. What is the potential energy of the boy when he is 1 m from the ground?
   (c) What is the potential energy of the boy when he lands on the ground?
3. A hiker walks up a mountain, 800 m above sea level, to spend the night at the top in the first overnight hut. The second day he walks to the second overnight hut, 500 m above sea level. The third day he returns to his starting point, 200 m above sea level.
   (a) What is the potential energy of the hiker at the first hut (relative to sea level)?
   (b) How much potential energy has the hiker lost during the second day?
   (c) How much potential energy did the hiker have when he started his journey (relative to sea level)?
   (d) How much potential energy did the hiker have at the end of his journey?

4.4 Kinetic Energy

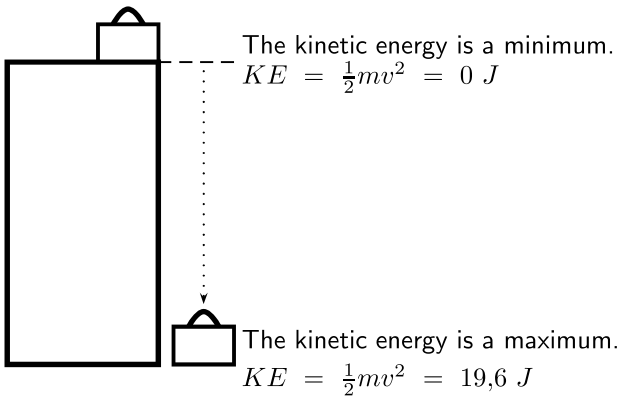
Definition: Kinetic Energy
Kinetic energy is the energy an object has due to its motion.

Kinetic energy is the energy an object has because of its motion. This means that any moving object has kinetic energy. The faster it moves, the more kinetic energy it has. Kinetic energy (KE) is therefore dependent on the velocity of the object. The mass of the object also plays a

role. A truck of 2000 kg, moving at 100 km·hr^{-1}, will have more kinetic energy than a car of 500 kg, also moving at 100 km·hr^{-1}. Kinetic energy is defined as:

$$KE = \frac{1}{2}mv^2 \tag{4.6}$$

Consider the 1 kg suitcase on the cupboard that was discussed earlier. When the suitcase falls, it will gain velocity (fall faster), until it reaches the ground with a maximum velocity. The suitcase will not have any kinetic energy when it is on top of the cupboard because it is not moving. Once it starts to fall it will gain kinetic energy, because it gains velocity. Its kinetic energy will increase until it is a maximum when the suitcase reaches the ground.



Worked Example 18: Calculation of Kinetic Energy

Question: A 1 kg brick falls off a 4 m high roof. It reaches the ground with a velocity of 8,85 m·s^{-1}. What is the kinetic energy of the brick when it starts to fall and when it reaches the ground?

Answer

Step 1 : Analyse the question to determine what information is provided

- The mass of the rock $m = 1$ kg
- The velocity of the rock at the bottom $v_{bottom} = 8{,}85$ m·s^{-1}

These are both in the correct units so we do not have to worry about unit conversions.

Step 2 : Analyse the question to determine what is being asked

We are asked to find the kinetic energy of the brick at the top and the bottom. From the definition we know that to work out KE, we need to know the mass and the velocity of the object and we are given both of these values.

Step 3 : Calculate the kinetic energy at the top

Since the brick is not moving at the top, its kinetic energy is zero.

Step 4 : Substitute and calculate the kinetic energy

$$\begin{aligned} KE &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(1\ \text{kg})(8{,}85\ \text{m}\cdot\text{s}^{-1})^2 \\ &= 39{,}2\,\text{J} \end{aligned}$$

4.4.1 Checking units

According to the equation for kinetic energy, the unit should be kg·m^2 ·s^{-2}. We can prove that this unit is equal to the joule, the unit for energy.

$$\begin{aligned}(\mathrm{kg})(\mathrm{m}\cdot\mathrm{s}^{-1})^2 &= (\mathrm{kg}\cdot\mathrm{m}\cdot\mathrm{s}^{-2})\cdot\mathrm{m} \\ &= \mathrm{N}\cdot\mathrm{m} \quad \text{(because Force (N) = mass (kg) × acceleration (m·s}^{-2}\text{))} \\ &= \mathrm{J} \quad \text{(Work (J) = Force (N) × distance (m))}\end{aligned}$$

We can do the same to prove that the unit for potential energy is equal to the joule:

$$\begin{aligned}(\mathrm{kg})(\mathrm{m}\cdot\mathrm{s}^{-2})(\mathrm{m}) &= \mathrm{N}\cdot\mathrm{m} \\ &= \mathrm{J}\end{aligned}$$

Worked Example 19: Mixing Units & Energy Calculations

Question: A bullet, having a mass of 150 g, is shot with a muzzle velocity of 960 m·s^{-1}. Calculate its kinetic energy?

Answer

Step 1 : Analyse the question to determine what information is provided

- We are given the mass of the bullet m = 150 g. This is not the unit we want mass to be in. We need to convert to kg.

$$\begin{aligned}\text{Mass in grams} \div 1000 &= \text{Mass in kg} \\ 150\ g \div 1000 &= 0{,}150\ \mathrm{kg}\end{aligned}$$

- We are given the initial velocity with which the bullet leaves the barrel, called the muzzle velocity, and it is v = 960 m·s^{-1}.

Step 2 : Analyse the question to determine what is being asked

- We are asked to find the kinetic energy.

Step 3 : Substitute and calculate

We just substitute the mass and velocity (which are known) into the equation for kinetic energy:

$$\begin{aligned}KE &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(150)(960)^2 \\ &= 69\,120\,\mathrm{J}\end{aligned}$$

Exercise: Kinetic Energy

1. Describe the relationship between an object's kinetic energy and its:
   (a) mass and
   (b) velocity
2. A stone with a mass of 100 g is thrown up into the air. It has an initial velocity of 3 m·s^{-1}. Calculate its kinetic energy
   (a) as it leaves the thrower's hand.
   (b) when it reaches its turning point.

3. A car with a mass of 700 kg is travelling at a constant velocity of 100 km·hr^{-1}. Calculate the kinetic energy of the car.

4.5 Mechanical Energy

Important: Mechanical energy is the sum of the gravitational potential energy and the kinetic energy.

Mechanical energy, U, is simply the sum of gravitational potential energy (PE) and the kinetic energy (KE). Mechanical energy is defined as:

$$U = PE + KE \tag{4.7}$$

$$\begin{aligned} U &= PE + KE \\ U &= mgh + \frac{1}{2}mv^2 \end{aligned} \tag{4.8}$$

4.5.1 Conservation of Mechanical Energy

The Law of Conservation of Energy states:

> Energy cannot be created or destroyed, but is merely changed from one form into another.

Definition: Conservation of Energy
The Law of Conservation of Energy: Energy cannot be created or destroyed, but is merely changed from one form into another.

So far we have looked at two types of energy: gravitational potential energy and kinetic energy. The sum of the gravitational potential energy and kinetic energy is called the mechanical energy. In a closed system, one where there are no external forces acting, the mechanical energy will remain constant. In other words, it will not change (become more or less). This is called the Law of Conservation of Mechanical Energy and it states:

> The total amount of mechanical energy in a closed system remains constant.

Definition: Conservation of Mechanical Energy
Law of Conservation of Mechanical Energy: The total amount of mechanical energy in a closed system remains constant.

This means that potential energy can become kinetic energy, or vise versa, but energy cannot 'dissappear'. The mechanical energy of an object moving in the Earth's gravitational field (or accelerating as a result of gravity) is constant or conserved, unless external forces, like air resistance, acts on the object.

We can now use the conservation of mechanical energy to calculate the velocity of a body in freefall and show that the velocity is independent of mass.

Important: In problems involving the use of conservation of energy, the path taken by the object can be ignored. The only important quantities are the object's velocity (which gives its kinetic energy) and height above the reference point (which gives its gravitational potential energy).

Important: In the absence of friction, mechanical energy is conserved and

$$U_{\text{before}} = U_{\text{after}}$$

In the presence of friction, mechanical energy is **not** conserved. The mechanical energy lost is equal to the work done against friction.

$$\Delta U = U_{\text{before}} - U_{\text{after}} = \text{work done against friction}$$

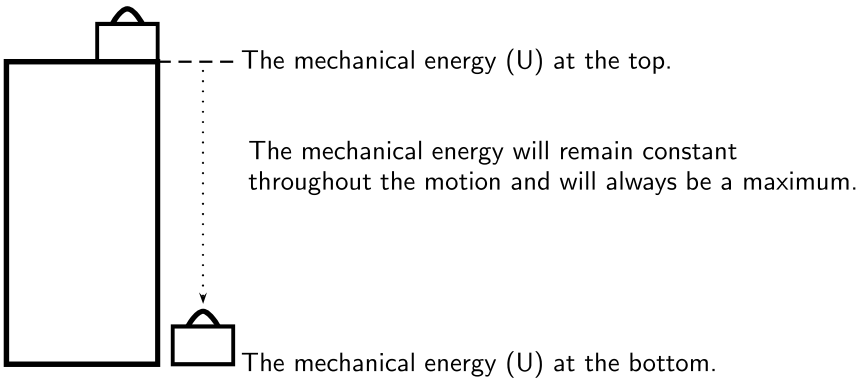
In general mechanical energy is conserved in the absence of external forces. Examples of external forces are: applied forces, frictional forces, air resistance, tension, normal forces.

In the presence of internal forces like the force due to gravity or the force in a spring, mechanical energy is conserved.

4.5.2 Using the Law of Conservation of Energy

Mechanical energy is conserved (in the absence of friction). Therefore we can say that the sum of the PE and the KE anywhere during the motion must be equal to the sum of the PE and the KE anywhere else in the motion.

We can now apply this to the example of the suitcase on the cupboard. Consider the mechanical energy of the suitcase at the top and at the bottom. We can say:



$$\begin{aligned}
U_{top} &= U_{bottom} \\
PE_{top} + KE_{top} &= PE_{bottom} + KE_{bottom} \\
mgh + \frac{1}{2}mv^2 &= mgh + \frac{1}{2}mv^2 \\
(1)(9{,}8)(2) + 0 &= 0 + \frac{1}{2}(1)(v^2) \\
19{,}6\ \text{J} &= \frac{1}{2}v^2 \\
39{,}2 &= v^2 \\
v &= 6{,}26\,\text{m}\cdot\text{s}^{-1}
\end{aligned}$$

The suitcase will strike the ground with a velocity of 6,26 m·s^{-1}.

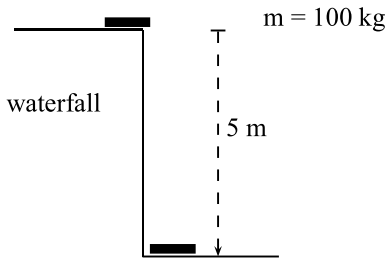
From this we see that when an object is lifted, like the suitcase in our example, it gains potential energy. As it falls back to the ground, it will lose this potential energy, but gain kinetic energy. We know that energy cannot be created or destroyed, but only changed from one form into another. In our example, the potential energy that the suitcase loses is changed to kinetic energy.

The suitcase will have maximum potential energy at the top of the cupboard and maximum kinetic energy at the bottom of the cupboard. Halfway down it will have half kinetic energy and half potential energy. As it moves down, the potential energy will be converted (changed) into kinetic energy until all the potential energy is gone and only kinetic energy is left. The 19,6 J of potential energy at the top will become 19,6 J of kinetic energy at the bottom.

Worked Example 20: Using the Law of Conservation of Mechanical Energy

Question: During a flood a tree truck of mass 100 kg falls down a waterfall. The waterfall is 5 m high. If air resistance is ignored, calculate

1. the potential energy of the tree trunk at the top of the waterfall.
2. the kinetic energy of the tree trunk at the bottom of the waterfall.
3. the magnitude of the velocity of the tree trunk at the bottom of the waterfall.



Answer

Step 1 : Analyse the question to determine what information is provided

- The mass of the tree trunk m = 100 kg
- The height of the waterfall h = 5 m.
  These are all in SI units so we do not have to convert.

Step 2 : Analyse the question to determine what is being asked

- Potential energy at the top
- Kinetic energy at the bottom
- Velocity at the bottom

Step 3 : Calculate the potential energy.

$$\begin{aligned} PE &= mgh \\ PE &= (100)(9{,}8)(5) \\ PE &= 4900 \text{ J} \end{aligned}$$

Step 4 : Calculate the kinetic energy.
The kinetic energy of the tree trunk at the bottom of the waterfall is equal to the potential energy it had at the top of the waterfall. Therefore KE = 4900 J.

Step 5 : Calculate the velocity.

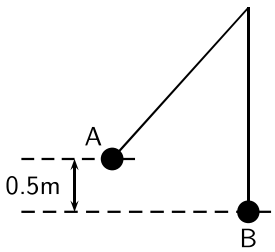
To calculate the velocity of the tree trunk we need to use the equation for kinetic energy.

$$\begin{aligned} KE &= \frac{1}{2}mv^2 \\ 4900 &= \frac{1}{2}(100)(v^2) \\ 98 &= v^2 \\ v &= 9{,}899... \\ v &= 9{,}90\,\mathrm{m}\cdot\mathrm{s}^{-1}\text{downwards} \end{aligned}$$

Worked Example 21: Pendulum

Question: A 2 kg metal ball is suspended from a rope. If it is released from point A and swings down to the point B (the bottom of its arc):

1. Show that the velocity of the ball is independent of it mass.
2. Calculate the velocity of the ball at point B.



Answer

Step 1 : Analyse the question to determine what information is provided

- The mass of the metal ball is $m = 2$ kg
- The change in height going from point A to point B is $h = 0{,}5$ m
- The ball is released from point A so the velocity at point, $v_A = 0$ m·s^{-1}.

All quantities are in SI units.

Step 2 : Analyse the question to determine what is being asked

- Prove that the velocity is independent of mass.
- Find the velocity of the metal ball at point B.

Step 3 : Apply the Law of Conservation of Mechanical Energy to the situation
As there is no friction, mechanical energy is conserved. Therefore:

$$\begin{aligned} U_A &= U_B \\ PE_A + KE_A &= PE_B + KE_B \\ mgh_A + \frac{1}{2}m(v_A)^2 &= mgh_B + \frac{1}{2}m(v_B)^2 \\ mgh_A + 0 &= 0 + \frac{1}{2}m(v_B)^2 \\ mgh_A &= \frac{1}{2}m(v_B)^2 \end{aligned}$$

As the mass of the ball m appears on both sides of the equation, it can be eliminated so that the equation becomes:

$$gh_A = \frac{1}{2}(v_B)^2$$

$$2gh_A = (v_B)^2$$

This proves that the velocity of the ball is independent of its mass. It does not matter what its mass is, it will always have the same velocity when it falls through this height.

Step 4 : Calculate the velocity of the ball

We can use the equation above, or do the calculation from 'first principles':

$$\begin{aligned}(v_B)^2 &= 2gh_A \\ (v_B)^2 &= (2)(9.8)(0{,}5) \\ (v_B)^2 &= 9{,}8 \\ v_B &= \sqrt{9{,}8}\ \mathrm{m \cdot s^{-1}}\end{aligned}$$

Exercise: Potential Energy

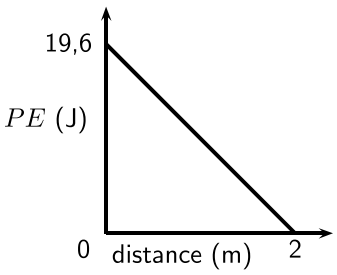
1. A tennis ball, of mass 120 g, is dropped from a height of 5 m. Ignore air friction.
   (a) What is the potential energy of the ball when it has fallen 3 m?
   (b) What is the velocity of the ball when it hits the ground?
2. A bullet, mass 50 g, is shot vertically up in the air with a muzzle velocity of 200 m·s^{-1}. Use the Principle of Conservation of Mechanical Energy to determine the height that the bullet will reach. Ignore air friction.
3. A skier, mass 50 kg, is at the top of a 6,4 m ski slope.
   (a) Determine the maximum velocity that she can reach when she skies to the bottom of the slope.
   (b) Do you think that she will reach this velocity? Why/Why not?
4. A pendulum bob of mass 1,5 kg, swings from a height A to the bottom of its arc at B. The velocity of the bob at B is 4 m·s^{-1}. Calculate the height A from which the bob was released. Ignore the effects of air friction.
5. Prove that the velocity of an object, in free fall, in a closed system, is independent of its mass.

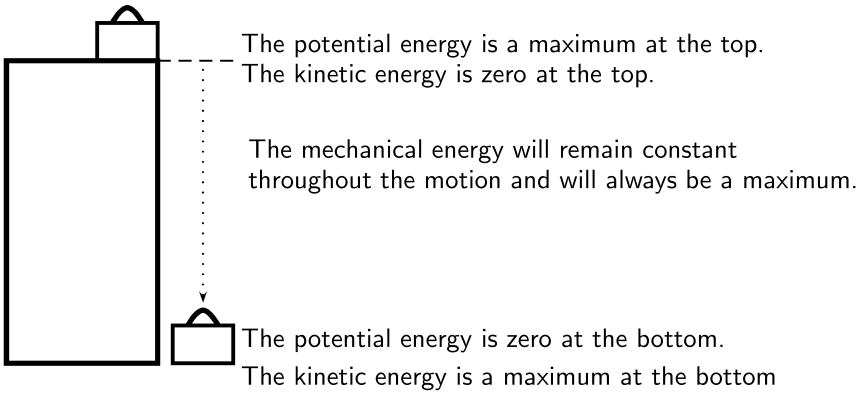
4.6 Energy graphs

Let us consider our example of the suitcase on the cupboard, once more.

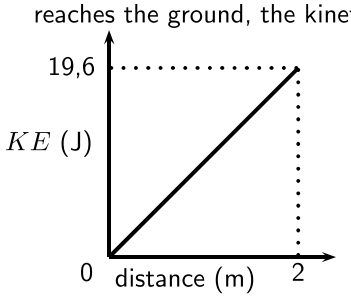
Let's look at each of these quantities and draw a graph for each. We will look at how each quantity changes as the suitcase falls from the top to the bottom of the cupboard.

- **Potential energy**: The potential energy starts off at a maximum and decreases until it reaches zero at the bottom of the cupboard. It had fallen a distance of 2 metres.

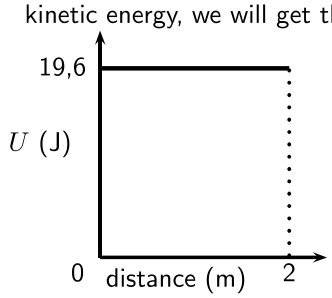




- **Kinetic energy**: The kinetic energy is zero at the start of the fall. When the suitcase reaches the ground, the kinetic energy is a miximum. We also use distance on the x-axis.



- **Mechanical energy**: The mechanical energy is constant throughout the motion and is always a maximum. At any point in time, when we add the potential energy and the kinetic energy, we will get the same number.



4.7 Summary

- Mass is the amount of matter an object is made up of.
- Weight is the force with which the Earth attracts a body towards its centre.
- A body is in free fall if it is moving in the Earth's gravitational field and no other forces act on it.
- The equations of motion can be used for free fall problems. The acceleration (a) is equal to the acceleration due to gravity (g).
- The potential energy of an object is the energy the object has due to his position above a reference point.
- The kinetic energy of an object is the energy the object has due to its motion.
- Mechanical energy of an object is the sum of the potential energy and kinetic energy of the object.
- The unit for energy is the joule (J).

- The Law of Conservation of Energy states that energy cannot be created or destroyed, but can only be changed from one form into another.
- The Law of Conservation of Mechanical Energy states that the total mechanical energy of an isolated system remains constant.
- The table below summarises the most important equations:

Weight	$F_g = m \cdot g$
Equation of motion	$v_f = v_i + gt$
Equation of motion	$\Delta x = \frac{(v_i+v_f)}{2}t$
Equation of motion	$\Delta x = v_i t + \frac{1}{2}gt^2$
Equation of motion	$v_f^2 = v_i^2 + 2g\Delta x$
Potential Energy	$PE = mgh$
Kinetic Energy	$KE = \frac{1}{2}mv^2$
Mechanical Energy	$U = KE + PE$

4.8 End of Chapter Exercises: Gravity and Mechanical Energy

1. Give one word/term for the following descriptions.
   (a) The force with which the Earth attracts a body.
   (b) The unit for energy.
   (c) The movement of a body in the Earth's gravitational field when no other forces act on it.
   (d) The sum of the potential and kinetic energy of a body.
   (e) The amount of matter an object is made up of.
2. Consider the situation where an apple falls from a tree. Indicate whether the following statements regarding this situation are TRUE or FALSE. Write only 'true' or 'false'. If the statement is false, write down the correct statement.
   (a) The potential energy of the apple is a maximum when the apple lands on the ground.
   (b) The kinetic energy remains constant throughout the motion.
   (c) To calculate the potential energy of the apple we need the mass of the apple and the height of the tree.
   (d) The mechanical energy is a maximum only at the beginning of the motion.
   (e) The apple falls at an acceleration of 9,8 m·s^{-2}.
3. [IEB 2005/11 HG] Consider a ball dropped from a height of 1 m on Earth and an identical ball dropped from 1 m on the Moon. Assume both balls fall freely. The acceleration due to gravity on the Moon is one sixth that on Earth. In what way do the following compare when the ball is dropped on Earth and on the Moon.

	Mass	**Weight**	**Increase in kinetic energy**
(a)	the same	the same	the same
(b)	the same	greater on Earth	greater on Earth
(c)	the same	greater on Earth	the same
(d)	greater on Earth	greater on Earth	greater on Earth

4. A man fires a rock out of a slingshot directly upward. The rock has an initial velocity of 15 m·s^{-1}.
   (a) How long will it take for the rock to reach its highest point?
   (b) What is the maximum height that the rock will reach?
   (c) Draw graphs to show how the potential energy, kinetic energy and mechanical energy of the rock changes as it moves to its highest point.

5. A metal ball of mass 200 g is tied to a light string to make a pendulum. The ball is pulled to the side to a height (A), 10 cm above the lowest point of the swing (B). Air friction and the mass of the string can be ignored. The ball is let go to swing freely.
   (a) Calculate the potential energy of the ball at point A.
   (b) Calculate the kinetic energy of the ball at point B.
   (c) What is the maximum velocity that the ball will reach during its motion?
6. A truck of mass 1,2 tons is parked at the top of a hill, 150 m high. The truck driver lets the truck run freely down the hill to the bottom.
   (a) What is the maximum velocity that the truck can achieve at the bottom of the hill?
   (b) Will the truck achieve this velocity? Why/why not?
7. A stone is dropped from a window, 3 metres above the ground. The mass of the stone is 25 grams.
   (a) Use the Equations of Motion to calculate the velocity of the stone as it reaches the ground.
   (b) Use the Principle of Conservation of Energy to prove that your answer in (a) is correct.